CARATHEODORY DISTANCE AND BOUNDED HOLOMORPHIC FUNCTIONS

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Let (X, A) be a ringed space with X hausdorff and D be a domain (open and connected) in X. Let Δ be the open unit disc in $\mathbf{C}$. Let $B = B(D)$ be the algebra of bounded A-holomorphic functions on D and B_1 be the family of A-holomorphic functions f of D into Δ with $\sup_{x \in D} |f(x)| = ||f||_D = 1$. We define the Carathéodory distance $c = c_D$ as follows: For $x, y \in D$

$$c(x, y) = \sup_{g \in B_1} \rho(g(x), g(y)),$$

where

$$\rho(z_1, z_2) = \log \frac{|z_2 - z_1| + |1 - z_1\bar{z}_2|}{\sqrt{(1 - z_1\bar{z}_1)(1 - z_2\bar{z}_2)}},$$

where $z_1, z_2 \in \Delta$. For $g \in B_1$ and $x' \in D$ set

$$f(x') = \frac{g(x') - g(x)}{g(x')\overline{g(x)} - 1},$$

then

$$c(x, y) = \sup_{f \in B_x} \left\{ \frac{1}{2} \log \frac{1 + |f(y)|}{1 - |f(y)|} \right\},$$

where

$$B_x = \{f \in B_1 ; f(x) = 0\}.$$

This distance c is a pseudo-distance on D and c is a distance if and only if $B(D)$ separates the points of D. We note that if $B(D)$ is a maximum modulus algebra then the distance c between two points of D is always finite and is a continuous function of D into $[0, \infty)$. D is a complete domain if every closed ball $\Delta(p, r) = \{x \in D; c(p, x) \leq r\}$, $p \in D$ and $r > 0$, is compact. D is boundedly holomorphic convex if for every compact subset K of D, $\hat{K}_B = \{x \in D; |f(x)| \leq ||f||_K$ for all $f \in B\}$ is compact. D is a domain of bounded holomorphy if there is a function in $B(D)$ which can not be continued holomorphically beyond D.

A point p in the closure $\bar{D}$ of D in X is called a point of finite distance if for each $x \in D$, $x \neq p$, there exists a neighborhood U of p in X and a finite positive number M such that $c(x, y) \leq M$ for all $y \in U_n D$. A point $p \in \bar{D}$ which is not

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