DIFFERENTIABILITY IN LOCAL FIELDS OF PRIME CHARACTERISTIC

CARL G. WAGNER

1. Introduction. In 1958 Mahler proved that every continuous p-adic function defined on the ring of p-adic integers is the uniform limit of an interpolation series of binomial form, and he exhibited a necessary and sufficient condition for such a function to be differentiable [2]. In [3] we showed that each continuous linear operator on the ring V of formal power series over a finite field (regarded as a vector space over that field) may be expanded in what may also be termed an interpolation series, and also characterized the differentiable operators. In [4] we dropped the linearity hypothesis of [3] and exhibited an interpolation series for each continuous function on V, and a sufficient condition for the differentiability of such a function. In the present paper we show (Theorem 4.1) that this condition is also necessary and thus obtain a complete characterization (Theorem 4.2) of differentiable functions of an "x-adic" variable.

2. Preliminaries. Denote by F the field of formal power series

$$\alpha = \sum_{i=-\infty}^{\infty} a_i x^i, \tag{2.1}$$

where the a_i are elements of the finite field $GF(q)$ of characteristic p, and all but a finite number of the a_i's vanish for $i < 0$. Let b be any real number such that $0 < b < 1$, and define an absolute value $|\ |$ on F by $|0| = 0$ and $|\alpha| = b^n$, where n is the least integer such that $a_n \neq 0$ for a nonzero α given by (2.1). As is familiar, F is complete with respect to the discrete non-archimedean absolute value $|\ |$ and, equipped with the metric topology induced by $|\ |$, F is a totally disconnected, locally compact topological field. In particular, polynomials over F give rise to continuous functions on F.

The valuation ring V of F consists of all formal power series of the form (2.1) where $a_i = 0$ for $i < 0$. V is compact and contains as a dense subring the ring $GF[q, x]$ of polynomials over $GF(q)$. Similarly, the quotient field of $GF[q, x]$, denoted $GF(q, x)$, is dense in F.

A polynomial $p(t)$ over $GF(q, x)$ is called *integral valued* if $p(m) \in GF[q, x]$ for all $m \in GF[q, x]$. A polynomial $p(t)$ over F is called *integral valued* (mod x) if $p(\alpha) \in V$ for all $\alpha \in V$. Since polynomials give rise to continuous functions

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