HOMOMORPHISMS OF GROUP ALGEBRAS

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The general problem considered here is how well do the various candidates for the group algebras of compact groups reflect the properties of the groups. In particular, we ask whether the existence of isometric or norm-decreasing homomorphisms between the group algebras reflect a homomorphism between the groups. The case of L^1-algebras and measure algebras for an arbitrary locally compact group has been considered by Greenleaf in [3], and the corresponding isomorphism problems are dealt with in [6], [9], [11], [12], [13], etc.

In this paper, only compact groups are considered, and the algebras $C(G)$, $L^p(G)$, $1 \leq p \leq \infty$, $p \neq 2$. In §1, it is shown that norm-decreasing monomorphisms and epimorphisms between C-algebras and between L^∞-algebras can only arise from epimorphisms of the group. In §2, similar results are proved for $L^p(G)$ with $1 \leq p < \infty$, $p \neq 2$ but the techniques developed here can only handle the case of *isometric* monomorphisms, and only norm-decreasing epimorphisms for $1 \leq p < 2$.

Finally, in §3, we give an application of these results giving conditions on the group algebras which ensure that a subgroup of a group is a semidirect summand.

For $i = 1$ or 2, G_i denotes a compact topological group with identity 1_i, and $\hat{G}_i$ denotes the set of continuous homomorphisms of G_i into the circle group Π. For a compact group G, $L^p(G)$ is a Banach algebra under convolution multiplication for $1 \leq p \leq \infty$. Let Soc (G) denote the set of those continuous functions on G whose translates generate a finite-dimensional vector space. Then Soc (G) is the socle of each of the algebras $L^p(G)$ and also of $C(G)$—the continuous functions on G. The algebra of bounded Borel measures on G will be denoted by $M(G)$, and normalized Haar measure by m_G.

§1. $C(G)$ and $L^\infty(G)$. Let G_1 and G_2 be compact groups, $\pi : G_1 \to G_2$ a continuous and open epimorphism and $\lambda \in \hat{G}_1$. Then $T : C(G_1) \to C(G_2)$, defined by

$$(Tf)(\pi x) = \int_{\ker \pi} \lambda(xy) f(xy)\, dy \qquad (x \in G_1\,, f \in C(G_1))$$

is a norm-decreasing epimorphism.

Also $S : C(G_2) \to C(G_1)$, defined by

$$(Sf)(x) = \lambda(x) f(\pi x) \qquad (x \in G_1\,, f \in C(G_2))$$

Received May 1, 1973. Revisions received January 11, 1974.