LOCALIZATION AND SELF-EQUIVALENCES

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Introduction. In this paper we apply techniques of localization [9] to the study of the group $Eq(X)$ of self-equivalences of a topological space X. The main theorem of Section 1 identifies $Eq(X)$ as a subgroup of $Eq(X_P) \times Eq(X_{P'})$, where P and P' are complementary sets of primes. In Section 2 we compute some examples.

1. All spaces herein will be of the homotopy type of simple CW-complexes with base point. All maps will preserve base points, and we will follow the custom of confusing maps and homotopy classes of maps when appropriate.

Our main reference for the notions of localization is [9]. In this respect we make the following notational conventions. P and P' will denote sets of primes such that $P \cup P' = \{\text{all primes}\}$ and $P \cap P' = \varnothing$. Z denotes the integers, Q the rational numbers, $Z_p = Z/pZ$ the integers modulo p, and $Q_P = \{r/s \in Q \mid p \in P \Rightarrow p \nmid s\}$. Q_P is the usual localization of the integers at the collection of primes P.

$Eq(X)$ is the set of equivalence classes of base point preserving homotopy equivalences of X, by the equivalence relation of base point preserving homotopy. This set becomes a group under the operation induced by composition of mappings. This group has been studied by several authors including Arkowitz–Curjel [1], Kahn [3], Olum [6], and Smallen [8].

The main theorem of this section provides a first step in computing $Eq(X)$ by means of the study of its localizations. The hope is that $Eq(X_P)$ will sometimes prove easier to compute than $Eq(X)$.

We have the following proposition.

PROPOSITION 1.1. *Let X be a finite CW-complex. Then there exists a monomorphism $\Phi : Eq(X) \to Eq(X_P) \times Eq(X_{P'})$.*

Proof. Consider the following homotopy commutative diagram such as (Figure 1) where γ_P, $\gamma_{P'}$, l_P, and $l_{P'}$ are the localization maps and $X_{(0)}$ is the localization of X at Q. The maps f_P and $f_{P'}$ exist by the universal property of localizations and are homotopy equivalences if f is one [5].

The map Φ given by $\Phi(f) = (f_P, f_{P'})$ is clearly well defined and if $\Phi(f) = \Phi(g)$, then $f = g$ by [2]. Thus Φ is one-to-one. (Note that a second proof of this last fact is found in [4] along with some discussion of a more general result.)

The next task is to identify the subgroup of $Eq(X_P) \times Eq(X_{P'})$ corresponding to $Eq(X)$. As in the proof of Proposition 1.1, there exist, via localization, maps

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