INFINITE ABELIAN GROUPS ARE HIGHLY TOPOLOGIZABLE

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1. Definitions and preliminary results. Let A be a set and let F be a countable set of algebraic operations on A. We will say that the algebraic structure (A, F) is *highly topologizable* if there are $2^{2^{|A|}}$ Hausdorff topologies on A, the maximum number possible, for which all of the operations in F are continuous.

While it has been known for some time (See, e.g., [5] or [2; p. 33].) that every infinite abelian group G admits nondiscrete, Hausdorff group topologies, it has only recently been proven by K.-P. Podewski [8] that any such G is highly topologizable. Podewski's result in fact applies to the class of "unrestricted" algebraic structure of which infinite abelian groups, infinite fields, and countable rings are all examples. The methods used generalize those of his paper [7] and, as would be expected for so general a result, are highly technical.

Our purpose here is to derive this result for infinite abelian groups in an easy way using well-known structure theory for groups and the previously known [6], [7] analogous result for infinite fields. One thereby gains the perhaps useful new insight that all of the difficult work in getting this result for groups and fields can be pushed to the field case. In fact, on the basis of [6] and [7], one sees that the difficult work can be pushed down as far as the countable fields.

Throughout this paper, all groups will be abelian and, unless stated otherwise, will be infinite.

LEMMA 1. *If H is a subgroup of a group G such that $|H| = |G|$ and if H is highly topologizable, then G is highly topologizable.*

Proof. A fundamental system of neighborhoods of zero for a group topology on H also serves as a fundamental system of neighborhoods of zero for a group topology on G. (See [1; p. 222, Proposition 1].)

Lemma 1 will allow us to turn our attention from G itself to nicely structured subgroups of G. In particular, it will be helpful to look at subgroups which are isomorphic to additive or multiplicative subgroups of fields.

Our next lemma states the known result on topologizing fields in a form suitable for relativizing to subgroups. For any topology $\mathfrak{T}$, $\mathfrak{U}$ will denote the set of $\mathfrak{T}$-neighborhoods of zero.

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