# COMPACT AND HILBERT-SCHMIDT INDUCED REPRESENTATIONS

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We consider the general question of when an induced representation is compact (CCR) or Hilbert–Schmidt.

We first show that an induced representation is CCR only if the original representation is CCR. We also give necessary and sufficient conditions for the $C^*$-algebra of a certain kind of transformation group to be CCR.

After discussing the concept of Hilbert–Schmidt representation, we obtain several results on when an induced representation is Hilbert–Schmidt. In conjunction with the normal subgroup case, we give necessary and sufficient conditions for product-convolution operators and induced representations of twisted group algebras to be Hilbert–Schmidt.

**1. Introduction.** In this paper we consider the general question of when an induced continuous unitary representation is compact, i.e., CCR, or Hilbert–Schmidt. The notion of CCR representation is well-established; however, this is not the case for Hilbert–Schmidt representation. We will discuss each in detail.

Let $H$ be a (second countable) locally compact group and $\pi$ a unitary representation of $H$ in the Hilbert space $\mathcal{H}$. Then $\pi$ is called CCR if the bounded operator $\pi(f) = \int_H f(x)\pi(x)\,dx$ is compact for each $f$ in $L^1(H)$. The group (or Banach*-algebra) $H$ is called CCR if each irreducible representation of $H$ is CCR. If $H$ is a closed subgroup of the locally compact group $G$, then we can form the induced representation $U^\pi$ of $G$ [9] and ask when $U^\pi$ is CCR. The author and others have investigated this problem extensively. Specifically, this problem was completely solved for the case when $H$ is normal in $G$ [3], [4]. Furthermore, in [14] the author showed that if $G/H$ is compact and $\pi$ is CCR, then $U^\pi$ is CCR. In Section 2 we show that in general $\pi$ CCR is a necessary condition for $U^\pi$ to be CCR (Theorem 2.2). As a consequence of these last two results, we are able to determine necessary and sufficient conditions for the $C^*$-algebra of a certain kind of transformation group to be CCR. Specifically, let $(G, X)$ be a (second countable) locally compact Hausdorff transformation group having the property that each stability group $G_x$, $x$ in $X$, is co-compact, i.e., $G/G_x$ is compact. Then the $C^*$-algebra [6], [7] corresponding to $(G, X)$ is CCR if and only if each $G_x$ is CCR (Theorem 2.4). This characterization is interesting in view of recent results by E. Gootman [7] on when this $C^*$-algebra is type I without the compactness assumptions (See Section 2.).

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