THREE INVARIANTS FOR COMPLEX MANIFOLDS

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Chern, Levine and Nirenberg generalized the harmonic length to get a seminorm $N(\Gamma)$ for the real homology groups of a complex manifold; $N(\Gamma)$ is a complex analytic invariant. Three such invariants $N(\Gamma)$, $N(\Gamma, \tilde{U})$ and $\tilde{N}(\Gamma, \tilde{U})$ are discussed here. If Γ has a complex analytic representative, then $N(\Gamma) = 0$. Product theorems are given which approximate $\tilde{N}(\Gamma, \tilde{U})$ when the manifold is a product of Dirichlet domains.

1. Introduction. One of the fundamental problems in the study of complex manifolds is finding whether two manifolds are analytically equivalent. Of course, one would like a naturally defined, computable, complete system of invariants of the complex structure to solve this classification problem. Aside from the very special case of compact Riemann surfaces, the general problem is quite open. In fact, there are very few invariants which have been found and calculated.

One starting place to look for analytic invariants is Riemann surfaces. If Γ is a class of curves on a Riemann surface W, then there is a natural measure of the harmonic capacity of Γ. This function is the *harmonic length* $N(\Gamma)$ studied by Landau and Osserman [7]. They set $N(\Gamma) = \sup_v \inf_{\gamma \varepsilon \Gamma} | \int_\gamma (\partial v/\partial N)\, ds|$, where $(\partial v/\partial N)\, ds$ is the normal differential of v and where the supremum is taken over all real harmonic functions $v : W \rightarrow (0, 1)$.

S. S. Chern, Harold I. Levine, and Louis Nirenberg realized it was more productive to write the integral as $\int_\gamma d^c v$, where $d^c v$ is the conjugate differential [2]. Then they extended the definition of $N(\Gamma)$ to homology classes $\Gamma \varepsilon H_p(M; R)$ of real C^∞ p-cycles γ on a complex manifold. This new function retains the following properties possessed by Landau and Osserman's harmonic length:

1. $N(\Gamma)$ is a seminorm on $H_p(M; R)$.
2. If $F : M \rightarrow M'$ is a holomorphic map of complex manifolds, then $N_M(\Gamma) \geq N_{M'}(F_* \Gamma)$ for all $\Gamma \varepsilon H_p(M; R)$.
3. Since $N(\Gamma)$ is defined using only the complex structure of M, it is a complex analytic invariant.

Thus, $N(\Gamma)$ has properties similar to Kobayashi's invariant metric [6].

In their paper, Chern, Levine, and Nirenberg introduced $N(\Gamma)$ and several similar invariants, proved their finiteness, and pointed out some basic properties. Unfortunately, these invariants are only seminorms; *a priori*, they could vanish identically. This paper and [4] are devoted to finding when the seminorms are true norms and to approximating them.

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