KERNELS OF MEASURES ON COMPLETELY REGULAR SPACES

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1. Introduction. A kernel is a function $\lambda : T \rightarrow M_x(S)$ from a topological space T into a space of measures $M_x(S)$ which is weak* continuous and bounded. This paper presents some results in the study of kernels and some corollaries in topological measure theory and duality theory. For example, a bounded linear operator $A: C(S) \rightarrow C(T)$ is defined by a kernel in the following manner. $A(f)(t) = \int f(s)\lambda(t, ds)$ and, under minimal conditions, such an operator defines a kernel. The kernel representation problem is to determine when A is weakly continuous (or equivalently, $A^*: M_x(T) \rightarrow M_x(S)$) and when

$$A^*\nu(E) = \int_T \lambda(t, E)\, d\nu \tag{1.1}$$

for $\nu \in M_x(T)$. Our results give conditions under which A is weakly continuous. The heart of the proof is the kernel representation of the adjoint A^*. A corollary states that in certain spaces of measures, the bipolar of a weak* compact set is weak* compact, thus showing that the Mackey and strong Mackey topologies agree for certain dual pairs $(C(S), M_x(S))$. In one space of measures the tool in the proofs of these results is a new type of additivity, L.S.C. additivity.

Kernels have been studied extensively by probabilists primarily in metric spaces where many results are consequences of the metric property. We continue the work of Sentilles [8] in the study of kernels from the point of view of the functional analyst. The kernel representation of line 1.1 in the case T and S are locally compact spaces is found in [8]. In [8] and [9] a kernel oriented study of operator continuity and compactness in the space of measures is made, resulting in applications to semigroups of operators on $C(S)$ [12] and Markov processes [10]. In this paper we extend the kernel representation to completely regular spaces, which are the most general topological spaces of interest in the study of spaces of continuous functions.

Section 2 of this paper contains definitions and notations used throughout the paper. Section 3 states some measure theoretic results which are used in the kernel representation theorems in Section 4. Section 5 contains some corollaries to the representation theorems which equate the Mackey and strong Mackey topologies.

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