SOME INVERSE RELATIONS

L. CARLITZ

1. Introduction. Gould and Hsu [2] have proved the following result.

THEOREM 1. *Let* $\{a_i\}$ *and* $\{b_i\}$ *be two sequences of complex numbers such that*

$$(1.1) \qquad a_i + b_i k \neq 0, \qquad i = 1, 2, 3, \cdots, k = 0, 1, 2, \cdots,$$

and put

$$(1.2) \qquad \psi(x, n) = \prod_{i=1}^{n} (a_i + b_i x).$$

The system of equations

$$(1.3) \qquad f(n) = \sum_{k=0}^{n} (-1)^k \binom{n}{k} \psi(k, n) g(k), \qquad n = 0, 1, 2, \cdots, N,$$

is equivalent to the system

$$(1.4) \quad g(n) = \sum_{k=0}^{n} (-1)^k \binom{n}{k} a_{k+1} + kb_{k+1}) \frac{f(k)}{\psi(n, k+1)}, \quad n = 0, 1, 2, \cdots, N,$$

where N *is a fixed positive integer or infinity.*

The principal object of the present paper is to prove the following q-analog of Theorem 1.

THEOREM 2. *Let* $\{a_i\}$ *and* $\{b_i\}$ *be two sequences of complex numbers, let* q *be an arbitrary complex number such that*

$$(1.5) \qquad a_i + q^{-k} b_i \neq 0, \qquad i = 1, 2, 3, \cdots, k = 0, 1, 2, \cdots,$$

and put

$$(1.6) \qquad \psi(x, n, q) = \prod_{i=1}^{n} (a_i + q^{-x} b_i).$$

The system of equations

$$(1.7) \qquad f(n) = \sum_{k=0}^{n} (-1)^k q^{\frac{1}{2}k(k-1)} \begin{bmatrix} n \\ k \end{bmatrix} \psi(k, n, q) g(k), \qquad n = 0, 1, 2, \cdots, N,$$

is equivalent to the system

$$(1.8) \qquad g(n) = \sum_{k=0}^{n} (-1)^k q^{\frac{1}{2}k(k+1)-kn} \begin{bmatrix} n \\ k \end{bmatrix} (a_{k+1} + q^{-k} b_{k+1}) \cdot \frac{f(k)}{\psi(n, k+1, q)}, \qquad n = 0, 1, 2, \cdots, N,$$

Received August 2, 1973. This research was supported in part by NSF grant GP-37927.