SMALL CONFIGURATIONS AND THEIR ARRAYS

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It is known that there is a bipartite $(n + 1)$-regular graph $G(n)$ of girth 6 on $2(n^2 + n + 1)$ vertices iff there are $n - 1$ m.o.l.s. of order n. When n is not a prime power, it is very hard to determine whether any $G(n)$ exist. None are known to exist, and many special theorems prove that large classes of $G(n)$ do not exist. If $T(n, k)$ is a generic term for a bipartite $(n + 1)$-regular graph of girth 6 on $2(n^2 + n + 1 + k)$ vertices, then it is known that there are $T(n, k)$ for every n and all large k. Since $T(n, 1)$ is so very nearly a $G(n)$, we mimic the construction of an affine plane from a projective plane and then examine the set of square arrays which are associated with $T(n, 1)$. Conditions on a set of arrays are determined which will produce a $T(n, 1)$ for each set of $n - 1$ arrays satisfying the conditions. The arrays are exhibited for $T(2, 1)$ and $T(3, 1)$.

1. Introduction. Let $n \geq 2$ be fixed in the discussion. Singleton [7] has shown that there is a bipartite $(n + 1)$-regular graph $G(n)$ of girth 6 on $2(n^2 + n + 1)$ vertices iff there is a projective plane of order n. In [3] Hall describes the equivalence of a projective plane of order n with a set of $n - 1$ mutually orthogonal latin squares or m.o.l.s. This equivalence is particularly useful in that a m.o.l.s. representation is more compact, easier to set in type and to proofread, and easier to manipulate than any other nonalgebraic form of representation. Very few algebraic representations are known, except for cyclic configurations, so the m.o.l.s. form is doubly important. Although the Desarguesian planes of prime power order were described in 1906 by Veblen [8] and although the first examples of non-Desarguesian planes of prime power order were given in the next journal volume (volume 9), it is still not known if there are any planes of composite order. Bruck, Ryser and Chowla [1], [2] have shown that planes, and therefore $G(n)$, fail to exist for many infinite classes of composite n.

Let $T(n, k)$ be a generic term for a bipartite $(n + 1)$-regular graph of girth 6 on $2(n^2 + n + 1 + k)$ vertices; then it is known [4] that for every n at least one $T(n, k)$ exists for all large enough k. Thus for every n there is a $K = K(n)$ such that there are $T(n, k)$ for all k exceeding K. It is not known in general whether the existence of a $T(n, k)$ implies the existence of $T(n, k + 1)$; so it is also of interest to know $K^* = K^*(n)$, the first k for which some $T(n, k)$ exists. Payne [6] has given several quite complex criteria for the interesting case $k = 1$ and has conjectured that k may equal 1 only if $n = 2$ or 3. He has shown that for many large classes, $T(n, 1)$ do not exist even though $T(n, 0)$ are known to exist.

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