DUALS OF FRECHET SPACES AND A GENERALIZATION OF THE BANACH–DIEUDONNÉ THEOREM

KALMAN BRAUNER

The purpose of this paper is to explore a class of locally convex spaces which can be characterized by the fact that they are duals of Frechet spaces with the topology of precompact convergence. In almost all respects these spaces, which I call dF spaces (cf. Definition 1.1), display the same properties as Frechet spaces except in a dual fashion. Perhaps this is not surprising since it is shown that the category of dF spaces is equivalent to the dual category of the category of Frechet spaces (cf. Proposition 3.2).

In Section 1, dF spaces are shown to be k-spaces, Krein–Smulian spaces, and semi-Montel spaces; they are shown to remain stable under the taking of closed subspaces, separated quotients, and countable direct sums. Also, in this section the relationship between dF spaces and the DF spaces of Grothendieck is discussed.

In Section 2 spaces of continuous linear maps from Frechet spaces to dF spaces and from dF spaces to Frechet spaces play the central role. With the topology of precompact convergence, it is shown that a set of continuous linear maps is precompact if and only if the members of the set are "uniformly compact" (cf. Proposition 2.3). In addition, a generalization of the Banach–Dieudonné theorem is proved where, rather than the scalars, the codomain space is allowed to be any dF space (cf. Corollary 2.7).

In Section 3 some category theoretic notions are investigated; in particular, the question of duality is looked at.

Finally, in Section 4 possible extensions of results of this paper are discussed.

The following notational conventions will be used throughout.

(i) $\mathbf{K}$ will denote the scalar field which will be either the complex numbers or the real numbers.

(ii) If E and F are locally convex spaces, then hom (E, F) will denote the vector space of all continuous linear maps from E to F, and $\text{hom}_p\ (E, F)$ will denote hom (E, F) with the topology of precompact convergence.

(iii) If E is a locally convex space, E^p will denote $\text{hom}_p\ (E, \mathbf{K})$. Normally parentheses will be omitted from iterated uses of the superscript "p". Thus $E^{pp} = (E^p)^p$.

Received March 17, 1973. This work is derived from the author's doctoral dissertation written at the University of California at Berkeley, 1972, under the supervision of Marc A. Rieffel, whose assistance and encouragement are gratefully acknowledged.