A HOPF ALGEBRA APPROACH TO THE KREIN DUALITY THEOREM

S. A. SELESNICK

For a compact group G let $R(G)$ denote the algebra of complex representative functions on G, that is, the algebra of coefficients of finite dimensional continuous unitary representations of G with operations inherited from $C(G)$, the algebra of continuous complex functions on G. The study of this object has a long history and its algebraic structure is well understood. Indeed, in 1949 M. G. Krein succeeded in giving an algebraic characterization of those algebras which can occur as an $R(G)$ [1], [5]. This characterization requires the existence of a special basis in the algebra consisting of matricial blocks and (among other things) the specification of a direct sum decomposition into other blocks of the Kronecker product of each pair of blocks (considered as matrices over the algebra) [1; 30.6]. The direct calculation of such decompositions for a concrete $R(G)$, however, presents formidable difficulties even in simple cases [1; §29]. More recently it has been realized that $R(G)$ has, further, the structure of a Hopf algebra and has been characterized as such by Hochschild (in the real case) [2] and by Hofmann [3], who found several equivalent formulations. Although these Hopf-algebraic characterizations are superficially more intrinsic than Krein's (no special basis being postulated) they do require extraneous elements of structure, notably, an integral-like "gauge" or functional on the algebra and the property that the set of hermitian complex homomorphisms of the algebra separates elements of the algebra. One observes that these requirements are absent from Krein's axioms.

This paper is an attempt to reconcile these two approaches to the converse Tannaka duality theorem. In Theorem 1 we give a set of axioms which characterize those Hopf algebras which can appear as an $R(G)$ but which do not require a priori the existence of an integral; instead, the existence of such a functional will be a consequence of algebraic results of M. E. Sweedler. (We note that axioms (1) and (2) of Theorem 1 specify a block basis precisely as in Krein's theorem and that the remaining axioms are analogous to those specifying the structure of a linear algebraic group over $\mathbf{C}$.) In Corollary 13 we show that this result leads at once to an alternative characterization of Krein algebras which imposes conditions directly upon the structure constants of the algebra in question but which avoids the explicit specification of the reducing matrices for Kronecker products of blocks.

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