EQUIVARIANT COBORDISM AND HOMOTOPY TYPE

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At the Second Conference on Compact Transformation Groups (University of Massachusetts, Amherst, 1971) Reinhard Schultz posed the question of whether equivariantly homeomorphic G-manifolds are necessarily equivariantly cobordant, G being a compact Lie group. This paper is concerned with the related question in which the assumed equivariant homeomorphism is replaced by an equivariant homotopy equivalence, a weakening of the hypotheses suggested by the well-known fact that unoriented cobordism class is a homotopy-type invariant.

In Section 1 we consider the special case in which the action of G is assumed to be free. Using standard techniques, we are able to prove that free G-manifolds having the same equivariant homotopy type are cobordant as free G-manifolds; this result holds for all compact Lie groups G. The next section considers the question for arbitrary actions of the cyclic group $\mathbf{Z}_2$; here Conner and Stong have shown that the result is true. We give a slightly more explicit proof of their result, which is primarily of interest for its implications concerning semifree actions of odd-order groups and finite abelian groups.

The results in Section 2 suggest that the basic difficulty in generalizing the result of Conner and Stong to other groups is the lack of a decent equivariant transversality theorem. So glaring is this deficiency that one should be led to conjecture that the result is, in general, false; in Section 4 we verify this conjecture by constructing, for each odd prime p, a family of counterexamples. The construction depends upon the discussion in Section 3 and the work of Olum on the homotopy-type of lens spaces.

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1. Free actions. Let G be a compact Lie group and denote by $\hat{\mathfrak{N}}_*^G$ the cobordism ring of (unoriented) manifolds with free G-action. If B_G is a classifying space for principal G-bundles, there is a well-known isomorphism

$$\hat{\mathfrak{N}}_*^G \cong \mathfrak{N}_*(B_G)$$

(with, possibly, a shift in dimension) given by classifying the orbit map; here $\mathfrak{N}_*(B_G)$ denotes the unoriented bordism of B_G . We should remark that a class in $\mathfrak{N}_*(B_G)$ is determined by its Conner-Floyd characteristic numbers [1], since we may choose a model for B_G in which the finite skeleta are honest manifolds.

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