SOME CHARACTERIZATIONS OF COMPLEX SPACE FORMS

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1. Introduction. Let M be a Kaehler manifold with complex structure J and Riemann metric g.

By a *plane section* we mean a 2-dimensional linear subspace of a tangent space. A plane section π is called *holomorphic* (respectively *anti-holomorphic*) if $J\pi = \pi$ (respectively $J\pi$ is perpendicular to π). The sectional curvature for a holomorphic (respectively anti-holomorphic) plane section is called holomorphic (respectively anti-holomorphic) sectional curvature.

A Kaehler manifold of constant holomorphic sectional curvature is called a *complex space form.* It is well known that a complex space form has constant anti-holomorphic sectional curvatures. Conversely, in Section 3 we shall prove the following theorem.

THEOREM 1. *Let M be a Kaehler manifold. If the anti-holomorphic sectional curvatures of M are constant and if* $\dim M \geq 3$, *then M is a complex space form.*

A Kaehler manifold M is said to satisfy the *axiom of holomorphic planes* (respectively the *axiom of anti-holomorphic planes*) if for each $x \in M$ and each holomorphic (respectively anti-holomorphic) plane π, there exists a 2-dimensional totally geodesic submanifold N such that $x \in N$ and $T_x(N) = \pi$.

Yano and Mogi [4] proved that a Kaehler manifold with the axiom of holomorphic planes is a complex space form.

In Section 4 we shall prove the following theorem.

THEOREM 2. *Let M be a Kaehler manifold. If M satisfies the axiom of anti-holomorphic planes and if* $\dim M \geq 3$, *then M is a complex space form.*

2. Preliminaries. In this section we shall give a brief summary of basic formulae.

Let M be a Kaehler manifold with complex structure J and Riemann metric g. We denote by R the curvature tensor field of M. Then we have

$$R(JX, JY) = R(X, Y) \tag{2.1}$$

$$R(X, Y)JZ = JR(X, Y)Z. \tag{2.2}$$

Let $K(X, Y)$ be the sectional curvature of M determined by orthonormal vectors X and Y. Then we have

$$K(JX, JY) = K(X, Y) \tag{2.3}$$

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