BANACH SPACES OF l^p VALUED HOLOMORPHIC MAPPINGS

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1. Introduction and definitions. A mapping $F = (f_j)$ defined on the unit disc of the complex plane and having values in an l^p space is said to be holomorphic if for every z_0 in the disc there is a continuous linear mapping $DFz_0 \in \mathcal{L}(\mathbf{C}, l^p)$ such that $||F(z) - F(z_0) - DFz_0\,(z - z_0)||_p = o\,(|z - z_0|)$.

Aron and Cima have shown in [1] that if $F = (f_j)$ is such an ℓ^p valued mapping and if f_j is holomorphic for each j, then the following are equivalent.

(a) F is holomorphic.
(b) F is continuous.
(c) F is weakly continuous.
(d) F is locally bounded.

For $1 \leq p < \infty$ an l^p valued holomorphic mapping F, for which $\log{}^+||F||_p$ has a harmonic majorant, has a Stoltz limit at almost every point of the unit circle [3].

We now define two distinct Banach space structures for l^p valued holomorphic mappings analogous to the H^p structure for complex valued ones. An l^s valued holomorphic mapping $F = (f_j)$ defined on the unit disc is said to be in $H_1^p(\Delta, l^s)$ if

$$_1||F|| = \sup_{r<1} \left(\frac{1}{2\pi}\int_0^{2\pi} ||F(re^{it})||_s^p\, dt\right)^{1/p} < \infty$$

and in $H_2^p(\Delta, l^s)$ if

$$_2||F|| = \sup_{r<1} \left(\sum_{j=1}^{\infty} \left(\frac{1}{2\pi}\int_0^{2\pi} |f_j(re^{it})|^p\, dt\right)^{s/p}\right)^{1/s} < \infty$$

where $1 \leq p < \infty$ and $1 \leq s < \infty$. It is easy to see that if $p = s$, then $_1||F|| = {}_2||F|| = (\sum_{j=1}^{\infty} ||f_j||^p)^{1/p}$.

Before beginning a discussion of these two spaces we note some properties of two related mixed norm L^P spaces which will be shown to possess closed subspaces isometrically isomorphic to $H_1^p(\Delta, l^s)$ and $H_2^p(\Delta, l^s)$. If N denotes the positive integers, the spaces

$$L^{(s,p)}(N \times [0, 2\pi]) = \left\{F : N \times [0, 2\pi) \to \mathbf{C} \;\middle|\; ||F||_{(s,p)} = \left(\frac{1}{2\pi}\int_0^{2\pi} \left(\sum_{j=1}^{\infty} |F(j, t)|^s\right)^{p/s} dt\right)^{1/p} < \infty\right\}$$

and

Received April 4, 1973.