ON THE GENERALIZED AND THE H-SHAPE THEORIES

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1. Introduction. Recently S. Mardešić [6] has given an extension of K. Borsuk's shape theory to include all topological spaces. T. Porter [9] then generalized this approach to obtain a "generalized shape theory". L. Rubin and the author [10] have also given an extension of Borsuk's shape theory to include all Hausdorff spaces. This extension will be called the H-shape theory. In this paper a category, called the H-shape category, is defined so that two Hausdorff spaces have the same H-shape if and only if they are equivalent objects in the H-shape category. Sum and products of Hausdorff spaces are then shown to be categorical sums and products, respectively, in the H-shape category. Finally, the generalized shape theory is discussed and a relationship that exists between the two theories is given.

A map $f : X \to Y$ is a continuous function from X to Y. An ANR is an absolute neighborhood retract for metrizable spaces. If $\mathfrak{T}$ is a category, the notation $X \in \mathfrak{T}$ means X is an object of $\mathfrak{T}$ and the notation $f \in \mathfrak{T}(X, Y)$ means f is a $\mathfrak{T}$-morphism from X to Y. The (compact) shape category given by Mardešić in [5] is denoted $\mathcal{C}$. The concept of the shape of a compact Hausdorff space, given by Mardešić and Segal in [7], is refered to as ANR-shape and denoted $\mathrm{Sh}_{\mathrm{ANR}}(X)$. Thus two compact Hausdorff spaces X and Y have the same ANR-shape, $\mathrm{Sh}_{\mathrm{ANR}}(X) = \mathrm{Sh}_{\mathrm{ANR}}(Y)$, provided there are compact shape maps $\underline{f} : X \to Y$ and $\underline{g} : Y \to X$ such that $\underline{g}\underline{f} = \underline{1}_X$ and $\underline{f}\underline{g} = \underline{1}_Y$, where $\underline{1}_X : X \to X$ is the identity compact shape map [5].

2. The category CS. In [10] L. Rubin and the author introduced a category CS which was used to give an extension of ANR-shape theory that includes all Hausdorff spaces. The objects of CS, called *CS-systems,* are direct systems $X^* = \{X_\omega , p_{\omega\omega'} , \Omega\}$ in the compact shape category $\mathcal{C}$. The morphisms, called *CS-morphisms*, are morphisms of direct systems $F = (f, \underline{f}_\omega) : X^* \to Y^* = \{Y_\lambda , q_{\lambda\lambda'} , \Lambda\}$ in the compact shape category $\mathcal{C}$. That is, $f : \Omega \to \Lambda$ is an increasing function and $\underline{f}_\omega : X_\omega \to Y_{f(\omega)}$, $\omega \in \Omega$, is a collection of compact shape maps such that if $\omega \leq \omega'$, then $q_{f(\omega)f(\omega')}\underline{f}_\omega = \underline{f}_{\omega'}p_{\omega\omega'}$, i.e., the diagram

$$\begin{array}{ccc} X_\omega & \xrightarrow{\underline{f}_\omega} & Y_{f(\omega)} \\ {\scriptstyle p_{\omega\omega'}}\downarrow & & \downarrow{\scriptstyle q_{f(\omega)f(\omega')}} \\ X_{\omega'} & \xrightarrow{\underline{f}_{\omega'}} & Y_{f(\omega')} \end{array}$$

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