OPEN MAPPINGS ON MANIFOLDS AND A COUNTEREXAMPLE TO THE WHYBURN CONJECTURE

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1. Introduction. In 1928 Stoilow proved that if f is a light open mapping between two 2-manifolds, then f is a discrete mapping. In 1951 Whyburn [12] proved that if f is a light open mapping of a 2-cell A onto A such that $f \mid \mathrm{Bd}\, A$ is a homeomorphism and $f(\mathrm{Int}\, A) = \mathrm{Int}\, A$, then f is a homeomorphism. In the same paper Whyburn conjectured that if in the above theorem A is an n-cell, then f is a homeomorphism. The main purpose of this paper is to show that the above theorems are false for light open mappings on higher dimensional manifolds.

The main theorems of this paper are the following.

THEOREM 1. *If M^k is a compact connected triangulated k-manifold, where $k \geq 3$, then there exists a monotone open mapping of M^k onto I^m.*

THEOREM 2. *If M^3 is a compact triangulated 3-manifold and $m \geq 3$, then there exists a light open mapping of M^3 onto I^m such that each point-inverse set is homeomorphic to the standard Cantor set.*

THEOREM 3. *If $m \geq 3$, then there exists a light open mapping f of I^m onto I^m such that if $p \in \mathrm{Bd}\, I^m$, then $f^{-1}(p) = \{p\}$, and if $p \notin \mathrm{Bd}\, I^m$, then $f^{-1}(p)$ is homeomorphic to the standard Cantor set.*

THEOREM 4. *If $m \geq 3$ and M^m is a triangulated m-manifold, then there exists a light open mapping f of M^m onto itself such that if $p \in \mathrm{Bd}\, M^m$, then $f^{-1}(p) = \{p\}$ and such that if $p \notin M^{(m-1)}$, the $m-1$ skeleton of M^m, then $f^{-1}(p)$ is homeomorphic to the standard Cantor set.*

Theorem 1 was announced by R. D. Anderson in 1956 [1]; however, he never published a proof. L. V. Keldys published a proof in Russian in 1957 [6]. Theorems 1 and 2 solve respectively Problems 41 and 42 proposed by Eilenberg [5]. Theorem 3 gives a counterexample to the Whyburn conjecture. Theorem 4, which follows immediately from Theorem 3, shows that the theorem of Stoilow is false for every m-manifold, $m \geq 3$. Also, since the function f in Theorem 4 does not have discrete point-inverse sets for each $y \in M^m$, Theorem 4 gives a counterexample to the conjecture [4] that every light open mapping from one triangulated manifold onto another is the uniform limit of simplicial open mappings.

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