ON MACKEY'S TENSOR PRODUCT THEOREM

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In [3] G. W. Mackey first introduced a definition of induced representations for separable locally compact groups and Hilbert spaces. (We will say that a group is separable if its topology has a countable base.) Using this definition, he proved four main theorems which he called the induction-in-stages theorem, the tensor product theorem, the subgroup theorem, and the intertwining number theorem. In [2] R. J. Blattner gave a definition of induced representations valid for arbitrary locally compact groups and Hilbert spaces (which is known to be equivalent to Mackey's original definition in the separable case) and, with this definition, extended the first of the above mentioned theorems to the nonseparable case. The purpose of this paper is to extend the tensor product theorem to the nonseparable case using Blattner's definition.

1. Blattner's definition of induced representations. The definition we give here is a reformulation of Blattner's definition by L. N. Argabright [1]. Let G be a locally compact group, H a closed subgroup, and L a unitary representation of H on the Hilbert space $\mathcal{H}(L)$. Let $C_0(G)$ be the space of all continuous complex-valued functions on G with compact support and let $C_0^+(G)$ be the set of nonnegative functions in $C_0(G)$. Let dx and dh denote the right Haar measure on G and H respectively, and let Δ and δ be the corresponding modular functions. By G/H we will mean the space of right cosets Hx of H in G and $\pi : G \to G/H$ will be the canonical projection of G onto G/H. We will denote $\pi(x)$ by $\dot{x}$.

Let $T : C_0(G) \to C_0(G/H)$ be defined by $\varphi \to T\varphi$, where $T\varphi(\dot{x}) = \int_H \varphi(hx)\, dh$. It is well known that T is surjective and, moreover, if $\psi \in C_0^+(G/H)$, then there is a function $\varphi \in C_0^+(G)$ such that $T\varphi = \psi$. Let $\mathcal{H}_0({}_GU^L)$ be the space of all continuous functions $f : G \to \mathcal{H}(L)$ with compact support modulo H, i.e., there is a compact set $K \subset G$ such that $\operatorname{Supp} f \subset HK$. For $f, g \in \mathcal{H}_0({}_GU^L)$ an easy computation shows that $\int_G (f(x), g(x))w_1(x)\, dx = \int_G (f(x), g(x))w_2(x)\, dx$ whenever $w_i \in C_0^+(G)$ and $Tw_i = 1$ on $\pi(K)$, $i = 1, 2$, where $K \subset G$ is compact and $\operatorname{Supp} f \cap \operatorname{Supp} g \subset HK$. Hence we can define an inner product on $\mathcal{H}_0({}_GU^L)$ by $(f, g) = \int_G (f(x), g(x))w(x)\, dx$, where w is as described above. The Hilbert space of the induced representation, $\mathcal{H}({}_GU^L)$, will then be the completion of $\mathcal{H}_0({}_GU^L)$ with respect to the above inner product. Later we will need to consider those functions $\epsilon_G(\varphi, v) \in \mathcal{H}_0({}_GU^L)$ defined by $\epsilon_G(\varphi, v)(x) = \int_H \delta(h)^{-\frac{1}{2}} \cdot \Delta(h)^{\frac{1}{2}}\varphi(hx)L_{h^{-1}}v\, dh$, where $\varphi \in C_0(G)$ and $v \in \mathcal{H}(L)$. Let $\mathcal{H}_{00}({}_GU^L)$ be the space of all such functions. In [2] Blattner shows that $\mathcal{H}_{00}({}_GU^L)$ is total in

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