STRICTLY CYCLIC OPERATORS

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It is shown that if the second commutant of a bounded operator on a Hilbert space has finite strict multiplicity, then the normal part of the operator is finite-dimensional and the completely nonnormal part has no nonzero summand whose norm equals its spectral radius.

1. Introduction. Let **H** be a complex Hilbert space, and let A be a subset of the algebra $\mathcal{B}(\mathcal{H})$ of all bounded linear operators on $\mathcal{H}$. Then $\mathcal{A}$ is said to have *finite strict multiplicity* if there exists a finite set $\Gamma = \{x_1, \cdots, x_n\}$ of vectors in $\mathcal{H}$ such that

$$\mathcal{A}(\Gamma) = \{A_1x_1 + \cdots + A_nx_n : A_1, \cdots, A_n \in \mathcal{A}\} = \mathcal{H}.$$

The minimum cardinality of all such sets Γ is called the *strict multiplicity* of $\mathcal{A}$. This concept is due to Herrero [3]. It generalizes the concept of strict cyclicity due to Lambert [4]. The set $\mathcal{A}$ is said to be *strictly cyclic* if it has finite strict multiplicity 1. A vector x such that $\mathcal{A}x = \mathcal{H}$ is called a *strictly cyclic vector* for $\mathcal{A}$. A vector x is called a *separating vector* for $\mathcal{A}$ if no two distinct operators in $\mathcal{A}$ agree at x.

An operator is said to have *finite strict multiplicity* n if the strongly closed algebra (containing the identity) it generates has this property. Note that this algebra is contained in the second commutant of the operator. (The *commutant* of a set $\mathcal{A}$ of operators is the set $\mathcal{A}'$ of all operators that commute with every operator in $\mathcal{A}$.) Therefore, the second commutant of an operator of finite strict multiplicity n has finite strict multiplicity at most n.

Our main result is that if the second commutant of an operator has finite strict multiplicty, then the normal part of the operator is finite-dimensional and the completely nonnormal part has no nonzero summand whose norm equals its spectral radius. An operator is *completely nonnormal* if it has no nonzero reducing subspace on which it is normal. An operator T is called *hyponormal* if $T^*T - TT^* \geq 0$ and *seminormal* if either T or T^* is hyponormal. We show that if $\mathcal{A}$ is a uniformly closed algebra which has a separating vector x such that $\mathcal{A}x$ is closed in $\mathcal{H}$, then $\mathcal{A}$ contains *no* seminormal operator with infinite spectrum. This extends a similar result for the class of subnormal operators obtained by W. Wogen and the author [1]. A corollary is that the same conclusion is valid if $\mathcal{A}$ is the commutant of a uniformly closed algebra of finite strict multiplicity, a result recently obtained by Lambert [5] using different methods. We apply these results to show that an infinite-dimensional sub-

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