A COHOMOLOGICAL CRITERION FOR REAL BOUNDARY POINTS

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1. Introduction. In the course of this paper, H_i will represent the *singular homology theory*, ${}_sH^i$ the *singular cohomology theory*, H^i the *A. S. cohomology theory*, and H^i_c the *A. S. cohomology theory with compact supports*; a bar above any of these symbols will mean that it is the *reduced theory*. G denotes an *arbitrary group* and F an *arbitrary field* in connection with *coefficients* for one of the above theories. X will be *a subset of* R^n, n-dimensional Euclidean space, and A *a compact subset* of X.

We will apply our results to *real normal clan acts* (S, X). By this we mean *a normal compact connected semigroup* S *with identity* 1, *a subset* $X \subseteq R^n$, and *a continuous function* $F : S \times X \to X$ *such that* $(s, (t, x)) = (st, x)$ for all s, $t \in S$ and $x \in X$. A semigroup act (S, X) induces a *quasi-order* $\leq$ on X in the following way. For $x, y \in X$, $x \leq y$ if and only if there exists $s \in S$ such that $x = sy$. A quasi-order $\leq$ is, of course, *a reflexive, transitive relation on* X and induces an equivalence relation $(\leq) \cap (\leq)^{-1}$. $K(S)$ will denote *the minimal ideal of* S.

In the case of topological semigroups and their actions on various state-spaces, an important role is played by the notion of *boundary point*. Except in Euclidean space the definition of a boundary point is open to discussion, and no altogether satisfactory notion has been advanced. In paracompact T_2-spaces, the leading candidate is the concept of *peripherality*. One may consult Lawson and Madison [4] for a general background to this question.

There is an example in [5] of an *irreducible hormos* (see [3]) acting on a state-space in R^3 so that a point *maximal* but not *minimal* in the quasi-order induced by the act is not peripheral, although it is in the boundary. D. P. Stadtlander conjectured that a point maximal but not minimal in the state-space of an action by a normal compact connected semigroup with identity must belong to the boundary if the state-space is in Euclidean space. To prove this, *linkage* suggests itself as a likely means. However, in order to be able to employ the strong homotopy properties of *A. S. cohomology*, it is desirable to formulate linkage criteria in the latter theory.

One uses also the fact that an equivalence class A (in R^n) of maximal points in the real state-space of an act by an irreducible hormos enjoys at each point x certain properties of a compact connected manifold, which have been collected in the following property.

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