A GENERAL POISSON LIMIT THEOREM OF PROBABILITY THEORY

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1. Summary. Let $A_1, A_2, \cdots, A_n$ be events on a given probability space and put $S_k(n) = \sum P(A_{i_1}A_{i_2} \cdots A_{i_k})$, $k \geq 1$, where the summation is for all choices $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ of the subscripts. Denoting by $p_k(n) = S_k(n)/\binom{n}{k}$ we observe that the numbers $p_1(n), p_2(n), \cdots, p_n(n)$ can be associated with a finite sequence $C_1, C_2, \cdots, C_n$ of exchangeable events so that $p_k(n) = P(C_1C_2 \cdots C_k)$, $k \geq 1$. The main result is that if, for n large, $C_1, C_2, \cdots, C_n$ can be extended into an infinite sequence of exchangeable events, then the sole assumptions $S_1(n) \to a > 0$, finite, and $S_2(n) \to \frac{1}{2}a^2$ imply that the limit distribution of the number ν_n of $A_1, A_2, \cdots, A_n$ which occur is Poisson with parameter a. This part is deduced from D. G. Kendall's limit theorem for exchangeable events. Applications are made to extreme value distributions by choosing $A_j = \{X_j \geq x\}$ for a given sequence $X_1, X_2, \cdots, X_n$ of random variables. The example given shows an essentially distinct feature of our limit result from the case of independent random variables and its known extensions.

2. Comparison with exchangeable events. For an arbitrary sequence $A_1, A_2, \cdots, A_n$ of events we put

$$S_k(n) = \sum P(A_{i_1}A_{i_2} \cdots A_{i_k}), \qquad k \geq 1, \tag{1}$$

where the summation is over all k-tuples

$$(i_1, i_2, \cdots, i_k) \quad \text{with} \quad 1 \leq i_1 < i_2 < \cdots < i_k \leq n.$$

Define

$$p_k(n) = S_k(n) \Big/ \binom{n}{k}. \tag{2}$$

Further, let ν_n be the number of the events $A_1, A_2, \cdots, A_n$ which occur. Our aim is to show that the distribution of ν_n can be determined from the special case when the A's are replaced by exchangeable events. We recall that a sequence $C_1, C_2, \cdots, C_n$ of events is called exchangeable if for any choice of the integers $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ the probabilities

$$\alpha_k = P(C_{i_1}C_{i_2} \cdots C_{i_k}) \tag{3}$$

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