ON $\Lambda_1(\alpha)$-NUCLEARITY

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In this paper it is shown that $\Lambda_1(\alpha)$-nuclearity coincides with pseudo-$\Lambda_1(\alpha)^\times$-nuclearity and that every $\Lambda_1(\alpha)$-nuclear map is of type $\Lambda_1(\alpha)^\times$. In addition to various examples and applications of these results, it is shown that the classes of $\Lambda_\infty(\alpha)$-nuclear spaces and the classes of $\Lambda_1(\beta)$-nuclear spaces mesh in a natural way. Finally, the relationship of permanence properties to stability is exhibited.

There has been much activity in recent years on the theory of λ-nuclearity, and in their Memoir, Dubinsky and Ramanujan [2] devote much attention to the case that λ is an infinite type power series space. In this paper we consider finite type power series spaces $\Lambda_1(\alpha)$ and give a characterization which shows that $\Lambda_1(\alpha)$-nuclearity is quite different from $\Lambda_\infty(\beta)$-nuclearity.

In Section 2 we show that the $\Lambda_1(\alpha)$-nuclear maps are the same as the pseudo-Λ_1-(α)-nuclear maps. In Section 3 we apply this to obtain diverse examples. In Section 4 we consider permanence properties of $\Lambda_1(\alpha)$-nuclearity and its relationship to stability.

1. Preliminaries. Most of the terminology is the same as in [2], with certain exceptions. In [2], $\Lambda(\alpha)$ refers only to infinite type power series spaces. Here we shall denote by $\Lambda_1(\alpha)$ the finite type power series spaces associated with α and by $\Lambda_\infty(\alpha)$ the infinite type power series spaces associated with α. Thus, for $(\alpha_n)_{n=0}^\infty$ such that $0 \leq \alpha_0 \leq \alpha_1 \leq \cdots$ and such that $\lim_n \alpha_n = \infty$

$$\Lambda_1(\alpha) = \{(t_n) : ||t||_k = \sum \left(\frac{k}{k+1}\right)^{\alpha_n} |t_n| < +\infty, \qquad k = 0, 1, \cdots\}$$

with Frechet topology generated by the norms $|| \ ||_k$.

$$\Lambda_\infty(\alpha) = \{(t_n) : ||t||_k = \sum k^{\alpha_n} |t_n| < +\infty, k = 0, 1, \cdots\}$$

with Frechet topology generated by the norms $|| \ ||_k$. It is well-known that $\Lambda_1(\alpha)$ is nuclear if and only if $\lim_n (\log n)/\alpha_n = 0$ and that $\Lambda_\infty(\alpha)$ is nuclear if and only if $\overline{\lim} (\log n)/\alpha_n < +\infty$ [5; 6.1.5].

For more facts of a general nature concerning the following concepts, refer to [2; p. 10ff]. Let E and F be normed linear spaces, and let λ be a sequence space. A linear map $T : E \to F$ is said to be λ-*nuclear* if $Tx = \sum \xi_n < x, a_n > y_n$ for all $x \in E$, where $\xi \in \lambda$, $(a_n) \in E'$, with $(||a_n||)_n \in l_\infty$, and $(y_n) \subseteq F$ is such that $(\langle y_n , b\rangle) \in \lambda^*$ for all $b \in F'$. Denote by $N_\lambda(E, F)$ the collection of λ-nuclear maps from E to F.

T is said to be *pseudo-λ-nuclear* if $Tx = \sum \xi_n\langle x, a_n\rangle y_n$ for all $x \in E$, where

Received January 22, 1973. Received revisions March 28, 1973.