ON STRONG EXTREME POINTS IN H^p

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1. Notation and definitions. Throughout this note **C** will denote the complex field, and $\mathbf{C}^n$ will be the cartesian product of n copies of **C**. The open disc in **C** is denoted by U; its boundary is the circle T. The unit polydisc U^n and the torus T^n are the subsets of $\mathbf{C}^n$ which are cartesian products of n copies of U and T respectively. The Banach space $H^\infty(U^n)$ consists of the analytic functions on U^n which are bounded. The Banach space $H^p(U^n)$ consists of those analytic functions f whose integral p-means are bounded,

$$\sup_{0<r_i<1} \frac{1}{2} \int_{T_n} |f(r_1e^{ix_1}, \cdots, r_ne^{ix_n})|^p \, dm_n(x) \leq M,$$

where m_n is normalized Haar measure on T^n. A function f in $H^\infty(U^n)$ is called an inner function if its radial limit values are equal to one in modulus a.e. on T^n. If X is a Banach space and Σ in its unit ball, the extreme points of Σ are those points of Σ which are not a proper convex combination of two distinct points of Σ. In [6; p. 113] R. McGuigan gives the following definition.

DEFINITION. Let X be a Banach space and let Σ be its unit ball. Let $||x - \Sigma|| = \inf \{||x - y|| : y \in \Sigma\}$. A point $x \in \Sigma$ is a strong extreme point of Σ if and only if for every $\alpha > 0$, $d(x, \alpha) > 0$ where

$$d(x, \alpha) = \inf_{||y||=1} \{\max [||(x + \alpha y) - \Sigma|| , ||(x - \alpha y) - \Sigma||]\}.$$

This definition is equivalent to the following criterion.
(A) A point $x \in \Sigma$, $||x|| = 1$, is a strong extreme point if and only if for each $\epsilon > 0$ there exists a $\delta > 0$ such that $\sup \{||x + y|| , ||x - y||\} \leq 1 + \delta$ implies $||y|| < \epsilon$.

2. Strong extreme points of H^∞. The extreme points of the unit balls of the $H^p(U)$ spaces are known [2]. A function f in $H^\infty(U)$ is an extreme point of the unit ball in $H^\infty(U)$ if and only if $|f(z)| \leq 1$ and

$$\int_{-\pi}^{\pi} \log [1 - |f(e^{ix})|] \, dx = -\infty.$$

The following theorem characterizes the strong extreme points of the unit ball of $H^\infty(U)$.

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