WEIGHTED SEQUENCES OF NONNEGATIVE INTEGERS

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1. Introduction. Carlitz [1] has defined $g_{j,k}(n)$ to be the number of sequences of nonnegative integers

$$\{a_1, a_2, \cdots, a_n\} \tag{1.1}$$

such that

$$|a_{i+1} - a_i| \leq 1, \qquad 1 \leq i \leq n-1, \tag{1.2}$$

and

$$a_1 = j, \qquad a_n = k. \tag{1.3}$$

His explicit formulas for $g_{j,k}(n)$ include

$$g_{j,k}(n+1) = \sum_{t=0}^{j} c(-t-1, j-t)[c(n+t, n+t-k) - c(n+t, n+t-k-2)], \tag{1.4}$$

where $c(n, k)$ is defined by

$$(1 + x + x^2)^n = \sum_{k=0}^{\infty} c(n, k)x^k \tag{1.5}$$

for n an arbitrary integer.

Letting $f_{j,k}(n; r, s)$ represent the number of sequences of nonnegative integers (1.1) that satisfy (1.3) and

$$-s \leq a_{i+1} - a_i \leq r, \tag{1.6}$$

where r and s are positive integers, the author [5] has shown that the generating function

$$\phi_{j,r,s}(x, y) = \sum_{n=0}^{\infty} \sum_{m=0}^{\min(n(r+s),\, j+nr)} f_{j,j+nr-m}(n+1; r, s)x^n y^m$$

can be expressed in terms of generalized binomial coefficients $c_{r+s}(n, k)$ defined by

$$\left(\sum_{e=0}^{r+s} x^e\right)^n = \sum_{k=0}^{\infty} c_{r+s}(n, k)x^k, \tag{1.7}$$

where n is an integer. For the cases $r = 1$ or $s = 1$ we have formulas which generalize (1.4), namely

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