HOMOGENEITY BY ISOTOPY FOR SIMPLE CLOSED CURVES

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1. Introduction. Suppose S^1 is a simple closed curve and $f(S^1)$ is an embedding of S^1 in E^3. The author, in [9], discusses the following homogeneity property for simple closed curves embedded in E^3. An automorphism is an onto homeomorphism.

DEFINITION 1.1. The simple closed curve S is *homogeneously embedded* or, alternatively, f is *homogeneous* if for any points p and q of S, there is an automorphism h of E^3 such that $h(S) = S$ and $h(p) = q$.

The author constructs examples of wild simple closed curves which are homogeneously embedded. The existence of homogeneously embedded 2-spheres is unknown. However, for the 2-sphere it is known that a homogeneity property involving isotopies implies a 2-sphere is tame. With this in mind we make the following definition.

DEFINITION 1.3. The embedding f or the simple closed curve S is *homogeneous by isotopy* if for any points p and q of S, there is an isotopy H of E^3 such that $H(S, t) = S$ for all t, $H(p, 0) = p$, and $H(p, 1) = q$.

Clearly, simple closed curves which are homogeneous by isotopy are homogeneously embedded. The question arises whether or not a simple closed curve which is homogeneous by isotopy is tame. In this paper we show that the most likely counterexamples, those known to be wild and homogeneously embedded, are not homogeneous by isotopy.

Notation and terminology. We use $\Pi_1(X)$ to denote the fundamental group of X and ∂X to be the boundary of X.

Suppose K_1 and K_2 are oriented knots. Then $K_1 + K_2$ is the oriented knot such that there is a 2-sphere R and an arc α in R such that

(1) $R \cap K = \{x, y\}, x \neq y$,

(2) α is an arc from x to y,

(3) $(\text{Int } R \cap K) \cup \alpha$ is the knot K_1,

(4) $(\text{Ext } R \cap K) \cup \alpha$ is the knot K_2.

The genus of the knot K is the smallest integer n such that K bounds an orientable surface of genus n. We will use the facts as shown in [5] that the genus of $K_1 + K_2$ is the genus of K_1 plus the genus of K_2 and that the genus of a nontrivial knot is greater than zero.

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