RIGHT GROUP AND GROUP CONGRUENCES ON A REGULAR SEMIGROUP

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1. Introduction. This paper investigates necessary and sufficient conditions on a regular semigroup in order that it have nontrivial right group (group) homomorphs. The term conventional semigroup is introduced to describe that class of regular semigroups S such that for the set of idempotents E_S, $cE_Sc' \subseteq E_S$ for each $c \in S$ and for each inverse c' of c. The minimum group congruence result obtained for this class generalizes that found for orthodox semigroups.

The relationship between E_S and the minimum right group (group) congruence on S is developed. The regular representation of S is found to be the maximum right group homomorphism if and only if E_S is a rectangular band. The minimum group congruence on S is found by using E_S to generate the minimum neat normal subsemigroup of S. Additional results are then given describing the kernel of the group homomorph in terms of E_S.

When a congruence ρ is such that S/ρ is *the* maximal homomorphic image of S of type C, as in [2; p. 18, Proposition 1.7], then S/ρ will be called the *maximum homomorphic image* of S of type C and ρ will be called the *minimum congruence* on S of type C. The phrases "right group congruence" and "group congruence" will be denoted by RGC and GC respectively. If such a congruence is minimum, it will be denoted by MRGC and MGC respectively.

The right-left duals of all results established will be taken for granted without further comment. For basic concepts, definitions, and terminology the reader is referred to Clifford and Preston [2]; in particular, $|S|$ denotes the cardinality of the set S, and the symbol $||$ will be used to indicate the end of a proof.

2. Regular semigroups. When S is regular and there is no danger of ambiguity, E will be used instead of E_S. The set of inverses of an element $c \in S$ will be denoted by $V(c)$.

Since any homomorphism θ of a regular semigroup S is regular, then by [2; p. 38, Theorem 1.27] any right simple image of a regular semigroup is a right group. If E is contained in a homomorphism class, then $S\theta$ is a group.

LEMMA 2.1 [2; p. 33, Exercise 4]. *A regular semigroup with exactly one idempotent is a group.*

If E is a subsemigroup of S, then S is called *orthodox*. Moreover, if E is a commutative semigroup, then S is called an *inverse* semigroup. The following

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