ON THE STRUCTURE OF FIBRATIONS

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Introduction. E. H. Brown, in his paper on twisted tensor products [1], developed the notion of an associated fibration to a given fibration. This construct, subsequently used by E. Dyer and D. S. Kahn in discussing fibrations and related spectral sequences [6], will be used to show that a family of fibrations over the subcomplexes of a cell complex can be extended to a fibration over that cell complex which is a weak colimit up to a fiber homotopy and distinguished with respect to any other weak colimit of the given family (see definition preceding Theorem 3.1).

To prove this result, vertical and horizontal colimit extensions of partially ordered families of fibrations are constructed. The latter extension affirmatively demonstrates the following replacement conjecture raised by E. Fadell as communicated by P. Tully [9]. Given a fiber space $\zeta = (X, p, B)$ with fiber $F = p^{-1}(b_0)$, $b_0 \in B$ and F' of the same homotopy type as F, there exist an open U such that $b_0 \in U \subset B$ and a fiber space $\zeta' = (X', p', B)$ such that ζ' is fiber homotopically equivalent to ζ, $p'^{-1}(b_0) = F'$, and $\zeta' \mid B - U = \zeta \mid B - U$.

The construction of the aforementioned distinguished weak colimit is then effected in the category whose objects are fibrations over locally contractible in the large pathwise connected base spaces and whose morphisms are fiber homotopic classes of maps between the total spaces and cofibrations between the bases.

1. Topological background. The definitions and results contained herein are formulated within the category of quasi-topological spaces and maps Q. The basic properties of this category are described by Spanier in [8]. In the forthcoming book by Dyer and Eilenberg [5] it is demonstrated that Q contains all necessary objects, maps, and properties needed in the exposition of this paper. For notational convenience the prefix "quasi" is dropped with the understanding that the constructions are confined to the category Q and all categories derived from it.

$\mathfrak{M}$ is the *mapping category* associated with Q, i.e., $p \in Ob\mathfrak{M}$ provided that p is a morphism in Q, and $(f, g) \in \mathfrak{M}(p, p')$ if and only if f and g are a pair of Q-morphisms such that

$$\begin{array}{ccc} X & \xrightarrow{f} & X' \\ \downarrow{\scriptstyle p} & & \downarrow{\scriptstyle p'} \\ B & \xrightarrow{g} & B' \end{array}$$

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