THE $\bar{\partial}$ EQUATION WITH POLYNOMIAL GROWTH ON A HILBERT SPACE

CHRISTOPHER J. HENRICH

To Professor D. C. Spencer on his sixtieth birthday

1. Introduction.

1.1. *Statement of problem.* Among overdetermined systems of partial differential equations, the inhomogeneous $\bar{\partial}$ equation is of special interest because of its relevance to the theory of holomorphic functions in several variables [5]. In this paper we describe an explicit solution of the equation $\bar{\partial}F = G$ in which F and G are forms of type $(0, q)$ and $(0, q + 1)$ having values with polynomial growth (and, of course, $\bar{\partial}G = 0$). The form G is defined on a Banach space $\mathbf{B}$, and F is defined on a Hilbert space $\mathbf{H}$ which is densely embedded in $\mathbf{B}$; for specifics on the pair $(\mathbf{H}, \mathbf{B})$ see Section 1.3 below.

The finite-dimensional case of this problem has already been solved by Skoda [12]. His solution is very similar to ours though slightly less explicit. If one expresses our solution in the form given by Equation (2.6.1) together with (1.4.11), then the first term (the integral with respect to ξ) corresponds to the first step of Skoda's solution. The other terms in (2.6.1) may be regarded as "correction terms"; Skoda achieves the same goal by a different route.

The analysis on infinite-dimensional spaces which we use is the work of several mathematicians, starting with Wiener. For a general treatment of Gaussian measures on infinite-dimensional spaces see [1], [3], [11]. In [2] Gross applies the Gaussian measure to solve problems involving the Laplace operator for a Hilbert space. We are dealing with only a first order operator and therefore can avoid some of his more delicate techniques. In fact, we use no property of infinite-dimensional Gaussian measures that is not strictly analogous to finite-dimensional measures.

As to the organization of this paper, Section 1.2 fixes some notation that is not quite standardized; the material is unoriginal, but it needs to be said. Section 1.3 summarizes facts we shall need concerning Gaussian measures. In Section 1.4 we state the main theorem with remarks about possible variations. The Formulae (1.4.4)–(1.4.12) are complicated and hard to understand without motivation; Part 2 explains the process by which they were written down, without attempting to give a rigorous proof of each step. The proof of the theorem occupies Parts 3 and 4; in Part 3 we show that our solution satisfies the desired bounds and in Part 4 that it solves the equation.

1.2. *Some notation.* If $\mathbf{B}$ is a locally convex vector space over the complex numbers, let $\bar{\mathbf{B}}$ denote the locally convex space coinciding with $\mathbf{B}$ as a topological

Received September 18, 1972.