THE C^1 EMBEDDING DIMENSION OF CERTAIN ANALYTIC SETS

JOSEPH BECKER AND JOHN STUTZ

Let V be a pure n dimensional analytic subset of U, a domain in C^m. We may define a sheaf $\mathfrak{D}_V^1$ of germs of real C^1 functions on V by taking for its stalk at $p \in V$, $\mathfrak{D}_{V,p}^1$, the germs of restrictions of C^1 functions defined in some neighborhood of p in U. The sheaf $\mathfrak{D}_V^1$ defines a ringed space structure on V, and associated with this structure at each $p \in V$ is a tangent space $T(V, \mathfrak{D}_{V,p}^1)$. This space determines the minimal dimensional space R^k in which a neighborhood of p in V may be C^1 imbedded. In [3] Bloom showed that $T(V, \mathfrak{D}_{V,p}^1)$ is a complex subspace of $T(V, \mathcal{O}_{V,p})$, the holomorphic tangent space, and if we set $d(p) = \dim_C T(V, \mathfrak{D}_{V,p}^1)$ and $h(p) = \dim_C T(V, \mathcal{O}_{V,p})$, then $d(p) = \dim_p V$ if and only if V is nonsingular at p. Bloom's results show that $d(p)$ is closely related to the analytic structure of V at p. However, the dependence as described in [3] is quite complicated. In particular, given $p \in V$ one gets no immediate idea when one has $d(p) = n + 1$ or $d(p) > n + 1$. In Section 1 of this note we provide this information for certain types of singularities. Our methods lead to the construction of examples of analytic sets on which there are C^1 weakly holomorphic functions h which are not holomorphic. (Weakly holomorphic functions are complex valued. By "C^1" we mean that their real and complex parts are C^1.) We obtain these examples without explicitly constructing functions and verifying their differentiability. As an example we show that if $V \subset U \subset C^{n+1}$ and p is a point of Sg V, the singular locus of V, at which V is irreducible and equisingular along Sg V, then such h exist if and only if the multiplicity of $\mathcal{O}_{V,p}$ is not 2. (For a discussion of equisingularity see [13] or [17].) Most of the results in Section 1 depend upon the fact that the dimension of L $C_5(V, p)$, the linear span of the tangent cone $C_5(V, p)$, is a lower bound for $d(p)$. In [3] Bloom showed that if p is an isolated point of Sg V, then $d(p) = \dim_C \mathrm{L}C_5(V, p)$. In Section 2 we show that for any pure dimensional analytic set V there is a negligible set $A \subset$ Sg V, of codimension greater than or equal to 1, such that if $p \in \text{Sg } V - A$, then $d(p) = \dim_C \mathrm{L}C_5(V, p)$. Here negligible is used in the technical sense described in [5].

1.

PROPOSITION 1. *Suppose* $p \in$ Sg V *and* $\dim_p$ Sg $V \leq n - 2$. *Then* $d(p) = n + 1$ *if and only if* $h(p) = n + 1$.

Received September 13, 1972. Partially supported by NSF Grant GU 3171.