KÄHLER MANIFOLDS AS REAL HYPERSURFACES

PATRICK J. RYAN

1. Techniques for decomposing hypersurfaces into products of the integral manifolds of the eigenspaces of the second fundamental form were developed by the author in [7] and [8]. In this paper we use these techniques to study a question recently posed and partially solved by Takahashi [9], namely the following.

Question. Which Kähler manifolds admit isometric immersions as real hypersurfaces in spaces of constant curvature? Such a hypersurface we will call a Kähler real hypersurface.

We first state the results of Takahashi.

**Theorem A.** Let $M^{2n}$ be a Kähler real hypersurface in a space $\mathbb{M}^{2n+1}$ of constant curvature $c$. Then the following hold.

(i) If $c = 0$, $M$ is flat if and only if the scalar curvature $s$ is zero.

(ii) If $n > 3$ and $c \neq 0$, then $M$ is flat and $c < 0$.

(iii) If $n = 2$ and $c \neq 0$, then $s \geq 0$. Also, $M$ is flat if and only if $s = 0$ and in this case $c < 0$.

Here we give a complete answer to the question for $c \neq 0$. Specifically we have the next theorem.

**Theorem B.** Let $M^{2n}$, $n > 1$, be a Kähler real hypersurface in a real space form of curvature $c \neq 0$. Then $M$ is an open subset of one of the following.

(i) $E^{2n}$ in $H^{2n+1}(c)$,

(ii) $S^2(c_1) \times S^2(c_2)$ in $S^4(c)$,

(iii) $S^2(c_1) \times H^2(c_2)$ in $H^4(c)$,

where all embeddings are the standard ones and $c_1$, $c_2$ and $c$ are real numbers of appropriate signs satisfying $c_1^{-1} + c_2^{-1} = c^{-1}$. If $M$ is assumed complete, then it is actually one of the above.

When $c = 0$, the problem is of a higher level of difficulty. There is a large class of examples, namely cylinders, built over surfaces in $E^3$. This class includes the hyperplanes and cylinders over plane curves. Furthermore, the work of Harle [3] suggests the existence of non-cylindrical embeddings of $S^2 \times E^{2n-2}$ in $E^{2n+1}$. A classification based on local considerations alone seems out of the question at present. However, the assumption of completeness allows us to prove the following theorem.

Received October 28, 1972. Research was supported by the National Science Foundation under grant GP-29662.

207