KÄHLER MANIFOLDS AS REAL HYPERSURFACES

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1. Techniques for decomposing hypersurfaces into products of the integral manifolds of the eigenspaces of the second fundamental form were developed by the author in [7] and [8]. In this paper we use these techniques to study a question recently posed and partially solved by Takahashi [9], namely the following.

Question. Which Kähler manifolds admit isometric immersions as real hypersurfaces in spaces of constant curvature? Such a hypersurface we will call a Kähler real hypersurface.

We first state the results of Takahashi.

THEOREM A. *Let M^{2n} be a Kähler real hypersurface in a space $\tilde{M}^{2n+1}$ of constant curvature c. Then the following hold.*

(i) *If $c = 0$, M is flat if and only if the scalar curvature s is zero.*

(ii) *If $n > 3$ and $c \neq 0$, then M is flat and $c < 0$.*

(iii) *If $n = 2$ and $c \neq 0$, then $s \geq 0$. Also, M is flat if and only if $s = 0$ and in this case $c < 0$.*

Here we give a complete answer to the question for $c \neq 0$. Specifically we have the next theorem.

THEOREM B. *Let M^{2n}, $n > 1$, be a Kähler real hypersurface in a real space form of curvature $c \neq 0$. Then M is an open subset of one of the following.*

(i) *E^{2n} in $H^{2n+1}(c)$,*

(ii) *$S^2(c_1) \times S^2(c_2)$ in $S^5(c)$,*

(iii) *$S^2(c_1) \times H^2(c_2)$ in $H^5(c)$,*

where all embeddings are the standard ones and c_1, c_2 and c are real numbers of appropriate signs satisfying $c_1^{-1} + c_2^{-1} = c^{-1}$. If M is assumed complete, then it is actually one of the above.

When $c = 0$, the problem is of a higher level of difficulty. There is a large class of examples, namely cylinders, built over surfaces in E^3. This class includes the hyperplanes and cylinders over plane curves. Furthermore, the work of Harle [3] suggests the existence of non-cylindrical embeddings of $S^2 \times E^{2n-2}$ in E^{2n+1}. A classification based on local considerations alone seems out of the question at present. However, the assumption of completeness allows us to prove the following theorem.

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