SPHERE BUNDLES OVER SPHERES AS LOOP SPACES MOD p

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We say that a space X has some property mod p if the localization of X at p has the property. Recent work of Curtis and Mislin [5] and Stasheff [11] has investigated when a sphere bundle over a sphere is an H-space mod p. Of particular importance are the spaces $B_n(p)$ described in [8] which occur in the mod p factorization of Lie groups [6], [8]. In [11] Stasheff has shown that most $B_n(p)$ are H-spaces mod p for p an odd prime and the question arises as to when $B_n(p)$ can be of the homotopy type of a loop space mod p. The main result of this note is to give necessary conditions on n and p for this to be true. The conclusion is that relatively few $B_n(p)$ are of the homotopy type of a loop space mod p.

For p an odd prime and n a positive integer the space $B_n(p)$ is an S^{2n+1}-bundle over $S^{2n+1+2(p-1)}$ classified by the generator of the p-primary part of $\pi_{2n+2(p-1)}(S^{2n+1})$. By [8], $H^*(B_n(p); Z/p)$ is an exterior algebra on generators x and y, where $\deg x = 2n + 1$, $\deg y = 2n + 1 + 2(p - 1)$ and $\mathcal{P}^1 x = y$. Work of Stasheff [11] has shown that for most pairs (n, p), $B_n(p)$ is an H-space mod p. In particular the following theorem and corollary of [11] are obtained.

THEOREM 1. *Let p be an odd prime and let n be a positive integer. If $2(4n + 2p) < 2pn + 2p - 2$, then $B_n(p)$ is an H-space* mod p.

COROLLARY 2. *If p is an odd prime greater than 5, then $B_n(p)$ is an H-space* mod p *for all positive integers n. If $p = 5$, then $B_n(p)$ is an H-space* mod p *for $n > 6$.*

Although most $B_n(p)$ are H-spaces mod p, the following theorem and corollary show that relatively few are of the homotopy type of a loop space mod p.

THEOREM 3. *Suppose p is an odd prime, n is a positive integer and $B_n(p)$ is of the homotopy type of a loop space* mod p. *If $p > 3$, then the following two conditions must be satisfied:*

(i) $2(p - 1) \equiv 0 \bmod n + 1$

(ii) $p^2 - 1 \equiv 0 \bmod p + n$.

If $p = 3$, then $n = 1$ or 5.

COROLLARY 4. *Suppose p is an odd prime and $n > 1$. If $B_n(p)$ is of the homotopy type of a loop space* mod p, *then $n^2 - n - 1 \geq p$.*

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