DOUBLY STABLE TWO-STAGE POSTNIKOV SYSTEMS

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1. A generalized Eilenberg-MacLane space is any countable Cartesian product $\times_i K(G_i , n_i)$ of Eilenberg-MacLane spaces in which each G_i is a finitely generated abelian group and $n_i \to \infty$ as $i \to \infty$. A two-stage Postnikov system ξ is a principal fibration whose base B and fibre F are connected generalized E-M spaces. Thus the system ξ is represented by a homotopy commutative diagram

$$\begin{array}{ccc} F & \xrightarrow{\text{id}} & B_0 \\ \downarrow{\scriptstyle i} & & \downarrow \\ E & \longrightarrow & PB_0 \\ \downarrow{\scriptstyle q} & & \downarrow \\ B & \xrightarrow{f} & B_0 \end{array} \tag{1}$$

in which B and B_0 are connected g E-M spaces, B_0 is simply connected, $PB_0 \to B_0$ is the path-space fibration, and E has the homotopy type of the fibre product of B and PB_0 over f.

Let p be a prime. When f is a "loop map", the mod p cohomology of E as a Hopf algebra over the Steenrod algebra A is determined by the structure of a submodule P of $H^*(E)$ which fits into a short exact sequence

$$0 \to R \xrightarrow{q^*} P \xrightarrow{i^*} X' \to 0$$

between a quotient module R of the cohomology of the base B and a submodule X' of the cohomology of the fibre F. A coproduct formula is established for elements of P and is used under a stronger stability assumption to attack the problem of computing the action of the Steenrod algebra.

The stability hypotheses will be explained in Section 3. Under the mildest of them, namely, when f is "stable mod p", the structure of $H^*(E)$ as an algebra over Z_p, and even over R, has been understood for over a decade [3], [4], [15], [16], [17], [18], [21]. If f is further assumed to be a loop map, then E is an H-space, and the coproduct structure of $H^*(E)$ has been determined by Harper and Schochet [9], who extended results obtained in a more restricted setting by Harper [7], [8] and independently by the author [6]. As Harper has pointed out [7], knowledge of the coproduct determines in principle the A-action on $H^*(E)$, up to unknown summands lying in a subgroup of the primitive elements of the image R of q^*, where $q : E \to B$ is the projection. Furthermore, using cochain operations, Kristensen and others have in principle computed the A-action without indeterminacy. (See, for instance, several articles by Kristensen, Madsen, and Kock in [23].) The main purpose of this paper is to remove some of the "in principle" from these computations, at least under the assump-

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