THE CONTAINING FRÉCHET SPACE FOR THE CLASS N+

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1. Introduction. Let D be the unit disk $\{|z| < 1\}$. A holomorphic function $f(z)$ in D is said to belong to the class N of functions of bounded characteristic if

$$(1.1) \qquad T(r, f) = \frac{1}{2\pi} \int_0^{2\pi} \log^+ |f(re^{i\theta})| \, d\theta = O(1) \quad \text{as} \quad r \to 1.$$

A function $f(z) \in N$ is factorized as [3; 24]

$$(1.2) \qquad f(z) = B(z; f)(S_1(z; f)/S_2(z; f))\Phi(z; f),$$

where $B(z; f)$ is the Blaschke product with respect to zero points of $f(z)$, $S_k(z; f)$, $k = 1, 2$, are the *singular inner functions* with no common factor, and $\Phi(z; f)$ is an *outer function for the class* N, i.e.,

$$(1.3) \qquad S_k(z; f) = \exp\left[-\int_0^{2\pi} \frac{e^{it} + z}{e^{it} - z} \, d\mu_f^k(t) \right]$$

with positive singular measures $d\mu_f^k$, $k = 1, 2$, and

$$(1.4) \qquad \Phi(z; f) = e^{i\gamma} \exp\left[\frac{1}{2\pi} \int_0^{2\pi} \frac{e^{it} + z}{e^{it} - z} \log \psi_f(t) \, dt \right]$$

with a real constant γ and a summable function $\log \psi_f(t)$. Then $\lim_{r\to1} |f(re^{it})| = \psi_f(t)$ holds for almost every $t \in [0, 2\pi)$.

A function $f(z) \in N$ is said to belong to the *class* N^+ if $S_2(z; f) \equiv 1$ [3; 25]. We have for $0 < p < q \leq \infty$ [3; 26]

$$(1.5) \qquad H^q \subset H^p \subset N^+ \subset N.$$

We have shown [9; Theorem 1] that the class N^+ forms an F-space in the sense of Banach [2; 51] with the distance function

$$(1.6) \qquad \rho(f, g) = \frac{1}{2\pi} \int_0^{2\pi} \log (1 + |f(e^{i\theta}) - g(e^{i\theta})|) \, d\theta$$

for $f, g \in N^+$.

The space N^+ with this metric (1.6) is not locally convex and is not locally bounded [9; corollary to Theorem 2].

For $0 < p < 1$, H^p also becomes an F-space, in the sense of Banach, which is not locally convex but is locally bounded, and Duren, Romberg, and Shields [4]

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