A BOUND FOR THE FIRST OCCURRENCE OF THREE CONSECUTIVE INTEGERS WITH EQUAL QUADRATIC CHARACTER

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1. Introduction and summary. Throughout k will be an integer greater than or equal to 2 and p a prime congruent to 1 (mod k). Let $C_k(p)$ denote the proper subgroup of the multiplicative group (mod p) consisting of the k-th powers (mod p). The $k - 1$ nonzero cosets formed with respect to $C_k(p)$ are called classes of k-th power non-residues and form with $C_k(p)$ a cyclic group of order k. Throughout S will denote the maximum number of consecutive elements in any of the k cosets.

Due to a well-known theorem of Brauer [1] for each positive integer m and sufficiently large p there exist a positive integer r and k-th power Dirichlet character modulo p such that

$$\chi(r) = \chi(r+1) = \cdots = \chi(r+m-1) = 1. \tag{1.1}$$

D. H. Lehmer and Emma Lehmer [7] denoted by $r(k, m, p)$ the smallest positive integer r satisfying (1.1). The finite number of primes for which r does not exist are called exceptional primes; all other primes are called non-exceptional. $\Lambda(k, m)$ is defined to be the least upper bound of $r(k, m, p)$, where the supremum is taken over all non-exceptional primes.

Jordan [6] weakened (1.1) by requiring only that

$$\chi(a) = \chi(a+1) = \cdots = \chi(a+m-1) \tag{1.2}$$

so that $a, \cdots, a+m-1$ are allowed to belong either to $C_k(p)$ or to exactly one of the $k - 1$ classes of k-th power non-residues. Letting $a(k, m, p)$ be the smallest positive integer a satisfying (1.2), Jordan defined $\Lambda^*(k, m)$ to be the least upper bound of $a(k, m, p)$, where the supremum again is taken over all non-exceptional primes.

Using the method of Lehmer and Lehmer [7] we see that it is easy to show $\Lambda(2, 3) = \infty$. In fact by preassigning character values for the positive integers as

$$\chi(n) = \left(\frac{n}{3}\right) = \begin{cases} 1 & \text{if } n \equiv 1 \pmod 3 \\ -1 & \text{if } n \equiv -1 \pmod 3 \end{cases} \tag{1.3}$$

and $\chi(3) = 1$ or -1 we obtain two assignments of character values which lead to an indefinite postponement of three consecutive quadratic residues and three consecutive quadratic non-residues. In view of the well-known theorem of Kummer in [4; Theorem 152] it is clear that for each fixed positive integer n

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