NORMAL BANDS OF COMMUTATIVE CANCELLATIVE SEMIGROUPS

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It is a classical result of Hewitt and Zuckerman [5] that every commutative separative semigroup is a semilattice of archimedean commutative cancellative semigroups and conversely. These semigroups have also been characterized by Schein [11] as subdirect products of commutative cancellative semigroups with a zero possibly adjoined. In addition, these are precisely the semigroups embeddable into semilattices of abelian groups. A different class of semigroups is obtained by taking all subdirect products of left or right zero semigroups with a zero possibly adjoined. This is precisely the class of all normal bands [8]. A construction of these semigroups was given by Kimura and Yamada [6].

In this paper we consider classes of semigroups which represent common generalizations of the above two classes or of some special cases thereof. In the course of our study, we find further interesting cases which do not arise in any of the two classes mentioned above. The most general class we consider is that in the title of the paper.

1. Definitions and summary. Let S be a semigroup. A congruence ρ on S is a $\mathcal{C}$-*congruence* if S/ρ belongs to a class $\mathcal{C}$ of semigroups. We will consider for $\mathcal{C}$ the following classes: semilattices, left and right zero semigroups, rectangular bands (in which case we speak of a *matrix* congruence), (left, right) normal bands (satisfying the identities $axy = ayx$, $xya = yxa$, $axya = ayxa$ respectively, in addition to $a^2 = a$). The least semilattice congruence on S will be denoted by $\mathfrak{N}$; its classes will be called $\mathfrak{N}$-classes. The resulting decomposition is the *greatest semilattice decomposition* of S. For precise definitions and an extensive discussion see the author's article [7]. A semigroup S is a *semilattice of semigroups belonging to a class* $\mathcal{C}$ if there exists a semilattice congruence of S all of whose classes belong to $\mathcal{C}$; a matrix, left zero band (instead of "left zero semigroup"), etc. of semigroups belonging to $\mathcal{C}$ is defined analogously. If $S = \bigcup_{\alpha \in Y} S_\alpha$, where Y is a semilattice and the semigroups S_α are the classes of the corresponding semilattice decomposition, and if there exist homomorphisms $\varphi_{\alpha,\beta} : S_\alpha \to S_\beta$ whenever $\alpha \geq \beta$, where $\varphi_{\alpha,\alpha}$ is the identity function on S_α, satisfying $ab = (a\varphi_{\alpha,\alpha\beta})(b\varphi_{\beta,\alpha\beta})$ if $a \in S_\alpha$, $b \in S_\beta$, then S is a *strong semilattice of semigroups* S_α.

A semigroup S is *weakly cancellative* (respectively *left separative*) if $ax = bx$ and $xa = xb$ (respectively $ab = a^2$ and $ba = b^2$) imply $a = b$; a commutative semigroup is *separative* if $ab = a^2 = b^2$ implies $a = b$. The direct product of a left zero semigroup, an abelian group and a right zero semigroup is a *rec-*

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