CANONICAL METRICS FOR CERTAIN CONFORMALLY EUCLIDEAN SPACES OF DIMENSION THREE AND CODIMENSION ONE

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1. In an earlier paper [1] the character of the metric of conformally Euclidean spaces of codimension one and having diagonal second fundamental tensor, i.e., spaces $\bar{C}_n^1$, was discussed for the cases $n \geq 4$. All solutions for these dimensions are such that $n - 1$ of the b_{ii} are equal, and in almost all cases the Codazzi equations are consequences of the Gauss equations (This is always true whenever τ, the rank of $[b_{ij}]$, is greater than or equal to 4; see [2].). Furthermore, it was pointed out that the imbedding (local and isometric) is unique except for the sign of $[b_{ij}]$, a feature which is automatically true if $\tau \geq 3$. Actually, one case was overlooked, namely, the case when $b_{ii} = 0$, $i = 2, \cdots, n$, and in fact, although $b_{ii} = 0$ gives a solution, as stated, it is not the only solution and therefore the imbedding here is not unique. This omission does not change the theorem in [1] and will be discussed below as it applies in the case $n = 3$ also.

This note presents the discussion of the spaces $\bar{C}_3^1$. Here $\tau \leq 3$, and the Codazzi equations must be solved in conjunction with the Gauss equations. As well as obtaining results which are identical with those for the higher dimensions, certain additional metric forms unique to the case $n = 3$ are found in which the b_{ii} are all distinct.

In the calculations which follow we assume the setting to be analytic, i.e., we are looking for analytic solutions for the first and second fundamental forms of $\bar{C}_3^1$.

2. The local and isometric imbedding conditions for C_3^1, i.e., a conformally Euclidean space of codimension one but with arbitrary second fundamental tensor, are the Gauss (2.1) and Codazzi (2.2) equations given by

$$r_{hijk} - (b_{hj}b_{ik} - b_{hk}b_{ij}) = 0, \tag{2.1}$$

$$b_{ij,k} - b_{ik,j} = 0, \tag{2.2}$$

where h, i, j, k run independently from 1 to 3.

For $\bar{C}_3^1$ we want a solution to these equations in which the second fundamental tensor is diagonal, i.e., $b_{ij} = 0$ if $i \neq j$.

Referred to a conformal coordinate system, the metric of a C_n can be written as

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