ORTHOCOMPACTINESS AND STRONG ČECH COMPLETENESS IN MOORE SPACES

PETER FLETCHER AND WILLIAM F. LINDGREN

In this paper it is shown that in a Moore space $(X, \mathfrak{I})$ the following properties are equivalent. (a) $(X, \mathfrak{I})$ admits a compatible non-archimedean quasi-metric; (b) $(X, \mathfrak{I})$ admits a compatible non-archimedean strong quasi-metric; (c) $(X, \mathfrak{I})$ is orthocompact. It follows that there exists a hereditarily realcompact Tychonoff space which is not orthocompact, and it follows that the existence of a separable perfectly normal Hausdorff space which is not orthocompact is consistent with the axioms of Zermelo-Frankel set theory (including the axiom of choice). The concept of strong Čech completeness is introduced. In a Tychonoff space strong Čech completeness implies Čech completeness. Every complete quasi-metric space is strongly Čech complete and every orthocompact strongly Čech complete space is a complete quasi-metric space. Every strongly Čech complete essentially T_1 space has a base of countable order so that a strongly Čech complete space is a complete Moore space if and only if it is θ-refinable. Consequently a space admits a compatible complete metric if and only if it is paracompact and strongly Čech complete. In a completely regular Moore space, Čech completeness and strong Čech completeness are equivalent properties; however, there exists a locally compact strongly Čech complete quasi-developable space which is not a Moore space.

1. Introduction. The origin and motivation of this paper are in the study of quasi-uniform spaces. The authors believe that there is an elementary relationship between complete quasi-developable spaces and complete quasi-uniform spaces with a countable base and that orthocompactness is an important link between these two concepts.

We introduce a powerful completeness property, which we call strong Čech completeness. Strong Čech complete spaces may be considered as generalizations of complete Moore spaces and, hence, of complete metric spaces. We show that Čech completeness and strong Čech completeness are equivalent properties in a completely regular Moore space and that strong Čech completeness implies Čech completeness in any space in which the latter concept is defined. Every essentially T_1 strongly Čech complete space has a G_δ-diagonal and a base of countable order so that every θ-refinable strongly Čech complete space is a complete Moore space. We do not know whether or not every strongly Čech complete essentially T_1 space is a complete quasi-developable space. Nevertheless, D. K. Burke has recently modified [19; Example 5I] and obtained

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