ON THE UNIQUENESS OF THE HEISENBERG COMMUTATION RELATIONS

MARC A. RIEFFEL

The purpose of this paper is to give an elementary proof of Mackey's form [9] of the theorem of Stone and von Neumann concerning the uniqueness of the Schrödinger representation of the Heisenberg commutation relations, whose main idea is to make precise the comment which Mackey makes on page 316 that the theorem "may be regarded as an infinite dimensional generalization of a classical result on the representations of full matrix algebras". The main part of our proof involves showing that any representation of the commutation relations is equivalent to a representation of an algebra of finite rank operators on an appropriate inner-product space. The second part of our proof then consists of applying the well-known theorem (for which we indicate a proof in Lemma 4) describing the representations of such an algebra (or, equivalently, of the algebra of compact operators). Other proofs can be found in [2], [7], [10], [14], [15]. For related results see [5] and [8] and the references mentioned there as well as [1] and [9].

An elementary proof of the uniqueness of the Heisenberg commutation relations was recently given by Segal and Kunze [15; Theorem 10.6] which has many points of close contact with the proof which we give. In fact, their proof can be interpreted to a large extent as being a mixture of the two parts of our proof. We feel that by separating these two parts and, in particular, by making explicit the role played by an algebra of finite rank operators, we contribute additional motivation and clarity to the proof.

It is well-known that the theorem on the uniqueness of the Heisenberg commutation relations is a special case of the imprimitivity theorem for induced representations of locally compact groups [10], [11], [14]. In a paper now in preparation (the main results of which were announced in [13]) we will show that one can associate to induced representations also an analogue of an algebra of finite rank operators, and we will use this fact to give a proof of the imprimitivity theorem for induced representations of groups and of C^*-algebras.

Let G be a locally compact group, and let $C_\infty(G)$ denote the C^*-algebra of continuous complex-valued functions on G which vanish at infinity and with pointwise operations. By a *unitary G-module* we will mean a Hilbert space W on which G acts by means of a strongly continuous unitary representation.

Received June 12, 1972. Revisions received September 22, 1972. The research for this paper was carried out while I was visiting at the University of Pennsylvania. I would like to thank the members of the Department of Mathematics of the University of Pennsylvania for their warm hospitality during my stay there. This research was partially supported by National Science Foundation grant GP-25082.