VITUSHKIN'S LOCALIZATION OPERATOR AND HOLOMORPHIC FUNCTIONS IN POLYDISCS

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In this paper Vitushkin's localization operator is extended to the case of bounded Borel measurable functions on C^n. This extended operator is used to study the behavior of certain classes of holomorphic functions in the open unit polydisc which have continuous extensions to subsets of the boundary. It is shown that holomorphic functions which extend continuously to appropriate subsets of the distinguished boundary can be uniformly approximated by holomorphic functions which extend holomorphically across corresponding boundary sets off the distinguished boundary. For a larger class of distinguished boundary sets a similar approximation theorem is obtained for the class of bounded holomorphic functions.

Another application of this extended operator gives a localization theorem for the n-fold tensor product of the algebra of bounded holomorphic functions in the open unit disc. This shows that the slice algebra fails to be the tensor algebra only if it fails locally near the distinguished boundary.

Introduction. The purpose of this paper is to give a several complex variable extension of Vitushkin's localization operator [9], [10], [4] and to apply it to the study of the class $H(U^n)$ of holomorphic functions in the open unit polydisc U^n. After obtaining this n-variable operator in Section 1, we use it in Section 2 to show that if $\alpha = (\alpha_1, \cdots, \alpha_n)$ belongs to the distinguished boundary T^n of U^n and if f in $H(U^n)$ extends continuously to $T^{k-1} \times \{\alpha_k\} \times T^{n-k}$ for $k = 1, \cdots, n$, then f can be uniformly approximated by holomorphic functions which extend holomorphically across α and $U^{k-1} \times \{\alpha_k\} \times U^{n-k}$ for $k = 1, \cdots, n$.

Also in Section 2 we view the algebra $H^\infty(U^n)$ of bounded holomorphic functions on U^n as a space of holomorphic vector-valued functions on U into $H^\infty(U^{n-1})$. From this observation it follows that if E is a subset of T such that the intersection of E with its boundary ∂E is countable, then those functions in $H^\infty(U^n)$ which extend holomorphically across $U^{k-1} \times E \times U^{n-k}$ are dense in those which extend continuously to $T^{k-1} \times E \times T^{n-k}$, $k = 1, \cdots, n$.

In Section 3 it is shown that the uniform closure $\bigotimes_{k=1}^{n}{}_\infty H^\infty(U)$ of the n-fold algebraic tensor product $\bigotimes_{k=1}^n H^\infty(U)$ is determined locally near the distinguished boundary T^n. This shows that the slice and tensor algebras are equal if and only if they agree locally near T^n [1].

1. Vitushkin's operator for $\mathbf{C}^n$. If $\varphi_1, \varphi_2, \cdots, \varphi_n$ are continuously differenti-

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