SETS IN $C(X)$ ANALYTICALLY EQUIVALENT TO THE OPEN BALL

H. E. WARREN

1. Introduction. The setting for this paper is the Lorch theory of analytic functions in commutative Banach algebras with identity. The basic ideas were put forth in [5]. See the remarks in [4] under "L-analytic functions" for a few main features of the theory and for comparisons with other kinds of generalized analytic functions. We shall use several of the specialized results of the Lorch theory which are contained in the papers [1] and [2] by B. W. Glickfeld. The object of the present paper is to further characterize the sets in $C(X)$ which are analytically equivalent to the open ball. An earlier result was given in [6]. A complete answer can be given for the sectionally disk equivalent sets discussed in Section 2. The characterization allows us to construct many nontrivial sets which are analytically equivalent to the open ball, among them a set which is not sectionally disk equivalent.

2. Sections of sets. Let C be the set of complex numbers, and let $D(z; r)$ be the open disk of radius r about $z \in C$. Write D for $D(0; 1)$.

The letter X will stand for a compact Hausdorff space. Write $C(X)$ for the continuous complex valued functions on X. The space $C(X)$ is a Banach algebra under pointwise algebraic operations and the norm $\|f\| = \sup \{|f(x)| : x \in X\}$. Let $B(f; r)$ be the open norm ball of radius r centered at $f \in C(X)$. Embed C in $C(X)$ by identifying a complex number z with the function identically equal to z. Write B for $B(0; 1)$.

As in the classical case, an analytic mapping Φ: $B(g; r) \rightarrow C(X)$ can be expanded in a power series $\Phi[f] = \sum g_n(f - g)^n$, where the g_n are fixed functions in $C(X)$. Conversely a power series $\sum g_n(f - g)^n$ defines an analytic function on $B(g; r)$, where $r = (\limsup \|g_n\|^{1/n})^{-1}$. The series converges on no larger ball centered at g.

An analytic mapping Φ: $B \rightarrow V \subset C(X)$ is an analytic equivalence of B onto V if Φ maps B one-to-one onto V and if Φ^{-1} is analytic on V. In searching for sets V which are analytically equivalent to B, we can make use of certain classical results by considering the sets $V_x = \{f(x) : f \in V\}$, where x is a point in X. Call V_x the section of V at x.

LEMMA 1. *If V is an open set in $C(X)$, then each section of V is open in C.*

Proof. If $z \in V_x$, then $z = f(x)$ for some $f \in V$. If V is open, there is an $\epsilon > 0$ with $B(f; \epsilon) \subset V$. Consequently $D(f(x); \epsilon) \subset V_x$ so that z is an interior point of V_x.

Received April 27, 1972.