# A NOTE ON PRINCIPAL CONSTRUCTIONS

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The purpose of this note is to prove that if $\bar{B}$ and $\bar{F}$ denote the (reduced) bar construction and cobar construction, then

(1) if $A$ is a connected associative DGA algebra, then $A \approx \bar{F}(\bar{B}(\bar{A}))$, the isomorphism of DGA algebras being in a strong sense, and

(2) if $C$ is a simply connected coassociative DGA coalgebra, then $C \approx \bar{B}(\bar{F}(C))$ as DGA coalgebras in a strong sense.

These two results are no surprise since in the geometric case we already know that (1)$'X \sim \Omega B(X)$ and (2)$'X \sim B(\Omega X)$, where $B$ stands for the classifying space and $\Omega$ stands for the loop space. The proofs of (1) and (2) are not completely obvious, however.

In the case of (1), for example, it is easy to construct $h$: $\bar{F}\bar{B}(A) \to A$ and $f$: $A \to \bar{F}\bar{B}(A)$. In fact $h$ is a homomorphism. On generators it has the form

$$h[[a_1]] = a_1$$

$$h[[a_1| \cdots |a_j]] = 0 \quad \text{if} \quad j \geq 2.$$

$f$ is given by $f(a) = [[a]]$ for all $a \in A$. $f$ is not homomorphism. However $f = f_1$ forms the initial map of a SHM map $\{f_1, f_2, \cdots, f_n, \cdots\}$, where

$$f_i(a_1 \otimes \cdots \otimes a_i) = [[a_1 |\cdots| a_i]].$$

It is direct to note that $hf = 1_A$ and $fh \simeq 1_{\bar{F}\bar{B}(A)}$. However, the difficult point is to show $fh$ is actually SHM-homotopic to 1.

This note in part contains proofs and details of some of the statements in Moore [10] and Stasheff and Halperin [12] and was written to answer a question of J. C. Moore and J. Stasheff.

**Preliminaries.** Let $K$ be a fixed commutative ring with unit. Let $ALG$ denote the category whose objects are connected associative DGA algebras over $K$ and whose maps are DGA algebra homomorphisms. Let $COALG$ denote the category whose objects are simply connected coassociative DGA coalgebras over $K$ and whose maps are DGA coalgebra homomorphisms.

We wish to define two more categories. First we need some notations and definitions.

Let $A \in ALG$. Let $A^n = A \otimes \cdots (n) \cdots \otimes A$ be the tensor product of $A$ with itself $n$ times. Define mappings

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