OPERATIONS IN MOD P CONNECTIVE K THEORY AND THE J HOMOMORPHISM

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Let **bu** denote the spectrum representing connective K theory and let $\mathbf{Z}_p$ denote both the cyclic group of prime order p and the corresponding co-Moore spectrum. It is known [3] that

$$\mathbf{bu} \wedge \mathbf{Z}_p \simeq \mathbf{k}_p \vee \textstyle\sum^2 \mathbf{k}_p \vee \cdots \vee \textstyle\sum^{2p-2} \mathbf{k}_p$$

for a spectrum $\mathbf{k}_p$. The spectrum $\mathbf{k}_p$ is somewhat easier to handle than $\mathbf{bu} \wedge \mathbf{Z}_p$ and it retains almost all the homotopy information of $\mathbf{bu} \wedge \mathbf{Z}_p$. In [3] a beginning is made in studying the ring of stable cohomology operations of the cohomology theory $k_p^*(\ )$. It is shown that there is an operation

$$P_1 : k_p^*(\ ) \to k_p^{*+2(p-1)}(\ )$$

which makes the following diagram commute

$$\begin{array}{ccc} k_p^*(X) & \xrightarrow[P_1]{} & k_p^{*+2(p-1)}(X) \\ \downarrow{\eta_p} & & \downarrow{\eta_p} \\ H^*(X; \mathbf{Z}_p) & \xrightarrow[P^1]{} & H^{*+2(p-1)}(X; \mathbf{Z}_p) \end{array}$$

for all reasonable spaces X, where η_p is induced by the natural augmentation to the Eilenberg–MacLane spectrum $\eta_p : \mathbf{k}_p \to \mathbf{K}(\mathbf{Z}_p)$ and P^1 denotes the first Steenrod reduced p-th power mod p. (Remember for $p = 2$, $P^1 = Sq^2$.) It is well known that for p odd, P^1 detects the element $\alpha_1 \in \pi_{2p-3}^s$ of the stable $2p - 3$ stem that generates the p-primary component of the image of the J homomorphism, and in fact [4], [5] it is known that this is all the homotopy that is detected by $\mathbf{Z}_p$-primary cohomology operations. The objective of the present note is to explore the analogous situation for the cohomology theory $k_p^*(\ )$ with particular reference to the J homomorphism and the operation P_1. Our main result may be stated in broad terms as follows.

THEOREM. *Let p be an odd prime. Then the operation P_1 detects the generator of the p component of the image of the J homomorphism in $\pi_{2t(p-1)-1}^s$ for all $t > 0$. For $p = 2$ the operation P_1 detects the generator of the 2 component of the image of the J homomorphism in π_{8t-1}^s and the μ family in π_{8t+1}^s for all $t > 0$.*

For example, when $p = 3$, P_1 detects the elements α_1, α_2, α_3', α_4, etc. of the 3, 7, 11 and 15 stems but fails to detect the element $\alpha_3 = 3\alpha_3' \in \pi_{11}^s$.

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