SOME STAR-INVARIANT SUBSPACES IN TWO VARIABLES

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Let U^n denote the polydisk: $|z_1| < 1, |z_2| < 1, \cdots, |z_n| < 1$ in C^n, and T^n its distinguished boundary: $|z_1| = |z_2| = \cdots = |z_n| = 1$. If μ is a positive measure on T^n, $L^p(d\mu)$ will denote its L^p spaces and $H^p(d\mu)$, $1 \leq p < \infty$, will denote the closure, in $L^p(d\mu)$, of the polynomials in $z_1, \cdots, z_n$. The Poisson integral of $d\mu$ is written

$$P[\mu] = \int_{T^n} P(s_1 - \theta_1) \cdots P(s_n - \theta_n)\, d\mu$$

where

$$P(\theta) = P_r(\theta) = (1 - r^2)/(1 + r^2 - 2r \cos \theta).$$

Let m and m_2 denote normalized Lebesgue measure on T and T^2 respectively:

$$dm = dt/(2\pi), \qquad dm_2 = ds\, dt/(2\pi)^2,$$

and let $B(z, w)$ be an inner function, i.e., let $B(z, w)$ be analytic, with $|B(z, w)| < 1$ in U^2, and $|B(e^{is}, e^{it})| = 1$ a.e. This paper is a study of the subspace

$$M^\perp = H^2(dm_2) \ominus BH^2(dm_2)$$

and of certain operators on $M^\perp$.

To be more specific, the function Re $(1 + B)/(1 - B)$ is positive and harmonic in U^2 and is, therefore, $P[\mu]$ for some (singular) measure μ on T^2. We determine explicitly (in Section 3 below) a unitary operator $\mathcal{U}$ which maps $M^\perp$ onto $H^2(d\mu)$. In Section 4 we consider the operators $S_1, S_2 : M^\perp \to M^\perp$ given by

$$S_1 f = Pzf, \qquad S_2 f = Pwf,$$

where $f \in M^\perp$ and where P is the projection of $H^2(dm_2)$ onto $M^\perp$. Sections 1 and 2 are devoted to the special type of measure representing $(1 + B)(1 - B)^{-1}$ and to its H^2 space.

The one variable analogue of this work, i.e., the case in which B is a function of one variable and $M^\perp$ is replaced by $H^2(dm) \ominus BH^2(dm)$, was done in my recent paper [3]. The main results of that paper generalize formally to two variables as we shall see. Most of the applications of those results (for example,

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