STRONGLY REGULAR MATRICES, ALMOST-CONVERGENCE, AND BANACH LIMITS

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1. Introduction. In this paper we prove three theorems concerning the relationship between the notion of almost-convergence introduced by Lorentz [8] and the set of strongly regular positive matrices. Lorentz proved that the space of almost-convergent sequences is not the intersection of the bounded convergence fields of a countable number of regular matrices. Theorem 1 states that it is exactly the intersection of the bounded convergence fields of *all* the positive strongly regular matrices. Theorem 2 makes precise the rough statement that the positive strongly regular matrices "generate" the set of Banach limits. Theorem 3 is an analogue of Theorem 1 for multipliers.

2. Preliminaries. We denote by l^∞ the Banach space of all bounded sequences $x = (x_0, x_1, \cdots)$ of real numbers. The shift operator S on l^∞ is defined by $(Sx)_n = x_{n+1}$.

Definition 1. A Banach limit φ is an element of $(l^\infty)^*$ satisfying the following conditions.

(1) $\varphi \geq 0$.
(2) $\varphi(u) = 1$ where $u_n = 1, n = 0, 1, \cdots$.
(3) $\varphi(Sx - x) = 0$ for all $x \in l^\infty$.

We denote by M the set of all Banach limits. It is well known that M is convex, is ω^*-compact and has cardinality 2^c [2].

Definition 2. $x \in l^\infty$ is almost-convergent to $a \in \mathbf{R}$ if $\varphi(x) = a$ for each φ in M. This is written F-lim $x = a$.

We denote by F the space of all almost-convergent sequences. F is a closed nonseparable subspace of l^∞, invariant under S [8]. Other characterizations of the space F can be found in [8]. A characterization of those sequences which are almost-convergent to 0 is given in [10].

Definition 3. If A is a regular matrix, we say that A is strongly regular provided that

$$\lim_n \sum_{m=0}^{\infty} |a_{n,m} - a_{n,m+1}| = 0.$$

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