SUBDIRECT DECOMPOSITION OF PŁONKA SUMS

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Introduction. Let K be an equational class of algebras of some fixed type τ without nullary operations. Following J. Płonka [5] an identity '$\mathbf{f} = \mathbf{g}$' in K is called *regular* if the set of variables occurring in the polynomial symbol $\mathbf{f}$ is the same as that in $\mathbf{g}$. Let $\mathbf{R}(K)$ denote the equational class of algebras defined by all the regular identities holding in K. If no algebra in K has an absorbing element (see §1 for definitions), the algebras in $\mathbf{R}(K)$ can be defined in terms of those in K by means of a construction due to J. Płonka [5] which we call the *Płonka sum*. In this paper we describe the subdirectly irreducible Płonka sums of members of K and thus, when no algebra in K has an absorbing element, the subdirectly irreducible members of $\mathbf{R}(K)$. Further results about $\mathbf{R}(K)$ will be presented in [3].

1. Notations and preliminaries. Throughout this paper K, L, M, etc. will denote equational classes of algebras of some fixed type τ without nullary operations (For these and other standard algebraic notions see [1].). By a *semilattice-ordered system* of algebras in K we mean a triple

$$\langle \mathfrak{J}, (\mathfrak{A}_i \mid i \in I), (\varphi_{ij} \mid i \leq j,\, i, j \in I)\rangle$$

where $\mathfrak{J} = \langle I; \vee\rangle$ is a join semilattice, $(\mathfrak{A}_i \mid i \in I)$ is a family of algebras in K indexed by the set I, and if $i \leq j$, $i, j \in I$, then φ_{ij} is a homomorphism from $\mathfrak{A}_i$ to $\mathfrak{A}_j$ satisfying the following two conditions.

(i) φ_{ii} is the identity mapping on A_i.

(ii) If $i \leq j \leq k$, then $\varphi_{ij}\varphi_{jk} = \varphi_{ik}$.

Given such a family of algebras in K, Płonka constructs an algebra of type τ in the following manner.

Let $A = \bigcup(A_i \mid i \in I)$, the disjoint union of the carrier sets of the algebras $\mathfrak{A}_i$. For an n-ary operation symbol $\mathbf{f}$ of τ we define its realization on A by setting

$$\mathbf{f}_{\mathfrak{A}}(x_1, \cdots, x_n) = \mathbf{f}_{\mathfrak{A}_j}(x_1\varphi_{i_1 j}, \cdots, x_n\varphi_{i_n j})$$

where $j = i_1 \vee \cdots \vee i_n$, $x_r \in A_{i_r}$, $r = 1, \cdots, n$, and $\mathbf{f}_{\mathfrak{A}_j}$ denotes the realization of $\mathbf{f}$ in the algebra $\mathfrak{A}_j$. We call the resulting algebra $\mathfrak{A} = \langle A; F\rangle$ the *Płonka sum* of the semilattice-ordered system $\langle \mathfrak{J}, (\mathfrak{A}_i \mid i \in I), (\varphi_{ij} \mid i \leq j,\, i, j \in I)\rangle$. The following results are due to Płonka (see [5] and [6] respectively).

(I) The equational class K is closed under the operation of taking Płonka sums if and only if $\mathbf{R}(K) = K$.

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