CONFLUENT IMAGES OF TREE-LIKE CURVES ARE TREE-LIKE

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Introduction. A metric topological space will be referred to as a *space* and a *map* will be a continuous function from one space to another space. By a *continuum* we will mean a compact connected space and a one-dimensional continuum will be a *curve*. A *graph* will be a one-dimensional polyhedron while a *tree* is a simply connected graph. Denote the standard n-sphere by S^n and the closed unit interval by I. A curve A is an *arc* if and only if there exists a homeomorphism f from I onto A. A curve X is *tree-like* (*arc-like*) provided if $e > 0$, there exists a tree (an arc) T and a map $f:X \to T$ such that if $t \in T$, then diam $(f^{-1}(t)) < e$.

If $g:X \to Y$ is a map such that g is homotopic to a constant map, then we will say $g \sim 0$. Otherwise, g non ~ 0 will mean that g is not homotopic to a constant map. The condition that a map $f:X \to Y$ is irreducible non-homotopic to a constant (f irr non ~ 0) means that f non ~ 0, but if B is a proper closed subset of X, then $f \mid B \sim 0$.

If X is a continuum and $f:X \to Y$ is a map, then f is *confluent* provided that whenever B is a subcontinuum of Y and F is a component of $f^{-1}[B]$ then $f[F] = B$.

The following question was asked by Lelek [10]. Suppose Y is a curve such that Y is the image of a tree-like curve under a confluent map. Is Y tree-like? Both the question and the affirmative answer to it that is presented in this paper are motivated by the next two theorems.

The first theorem is by Case and Chamberlin [3] and provides a useful characterization of tree-like curves.

THEOREM 1. *If Y is a curve, then Y is tree-like if and only if whenever g is a map of Y onto a finite linear graph then $g \sim 0$.*

The other theorem is due to Lelek [9] and is stated in the form that we will use it.

THEOREM 2. *If $f:X \to Y$ is a confluent map of a continuum onto a continuum and $g:Y \to S^1$ is a map such that $g \circ f \sim 0$, then $g \sim 0$.*

Let X be a continuum. Then X is *decomposable* provided there exists a pair of proper subcontinua of X whose union is X and is *indecomposable* if it is not decomposable. Also, X is *hereditarily decomposable* if each subcontinuum of X is decomposable.

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