THE DUAL SPACE OF H^p OF THE POLYDISC FOR $0 < p < 1$

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1. Introduction. A function f, analytic in the unit disc, is said to be in H^p, $0 < p < \infty$, if

$$M_p(r; f) = \left(\frac{1}{2\pi}\int_0^{2\pi} |f(re^{i\theta})|^p \, d\theta\right)^{1/p}$$

is bounded for $0 \leq r < 1$. Hardy and Littlewood proved

$$\int_0^1 (1 - r)^{\lambda a - 1} M_q^\lambda(r; f) \, dr < \infty$$

for all f in H^p, where $0 < p < q \leq \infty$, $\lambda \geq p$, and $a = 1/p - 1/q$. (See [5] or [1; 87].) As an application of a measure theorem for the polydisc, Duren and Shields [4] have extended this inequality to the polydisc in the case $\lambda > p$. Independent of their work, we extend this inequality to the polydisc for $\lambda \geq p$.

We use the generalized Hardy–Littlewood inequality to characterize the continuous linear functionals on H^p of the polydisc for $0 < p < 1$. Duren, Romberg, and Shields [2; Theorem 1] have shown in the case of the disc that each such functional ϕ has the form

$$\phi(f) = \lim_{r \to 1} (2\pi)^{-1} \int_0^{2\pi} f(re^{i\theta}) g(e^{-i\theta}) \, d\theta, \qquad f \in H^p,$$

for some analytic function g satisfying

$$|g^{(m)}(z)| \leq C(1 - |z|)^{1/p - m - 1}, \quad \text{where} \quad 1/(m + 1) < p \leq 1/m.$$

We shall give a natural generalization of this theorem. In the course of the proof we also extend various results of Hardy and Littlewood to the polydisc.

2. Preliminaries. Before proceeding we need to introduce some notation. Let

$$\Delta^n = \{z = (z_1, \cdots, z_n) : |z_j| < 1, \quad j = 1, \cdots, n\}$$

denote the polydisc in C^n, the space of n complex variables. Let

$$T^n = \{w \in C^n : |w_j| = 1, \quad j = 1, \cdots, n\}$$

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