LATTICE PROPERTIES OF $\mathcal{L}$ (E, F)

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1. Introduction and results. When E and F are vector lattices the space $\mathcal{L}(E, F)$ of continuous linear mappings from E to F need not be a vector lattice for the order determined by the cone of positive continuous linear mappings from E to F. For example, consider the space $\mathcal{L}(l^2, l^2)$. (See [2; Chapter 4, 3.3].) In this paper we obtain three new sufficient conditions for $\mathcal{L}(E, F)$ to be a vector lattice. The proofs have a common thread in the use of a seminorm inequality. We also obtain a sufficient condition for the subspace of bounded linear mappings to be a vector lattice and a condition for which $\mathcal{L}(E, F)$ is a topological vector lattice when properly topologized.

The notation and terminology for ordered vector spaces is as in [2]. We recall the definitions of T^+ and $|T|$. Let E, F be vector lattices and let $T : E \rightarrow F$ be linear. For $x \geq \theta$, the zero element, in E let $T^+x = \sup \{Ty \mid \theta \leq y \leq x\}$ if the sup exists; and for general $x \in E$ let $T^+x = T^+x^+ - T^+x^-$. Also $|T| : E \rightarrow F$ is defined by $|T|\ x = \sup \{\sum_{i=1}^n |Tx_i| \mid \sum_{i=1}^n x_i = x, x_i \geq \theta\}$ for $x \geq \theta$. In order to prove that $\mathcal{L}(E, F)$ is a vector lattice it is enough to show that either T^+ or $|T|$ exists for each $T \in \mathcal{L}(E, F)$. Our first theorem is an improvement of a result due to Krengel [1] (also see [2; 174, 3.8]).

THEOREM 1. *Let E be a normed vector lattice such that the norm is additive on the cone and let F be a vector lattice which is boundedly and locally order complete. Then $\mathcal{L}(E, F)$ is a vector lattice.*

Our proof uses a seminorm inequality stated in the following lemma. All proofs are given in Section 2.

LEMMA 1. *If F is a locally convex vector lattice which is boundedly and locally order complete, then for any lattice seminorm p_β*, i.e., $|x| \leq |y|$ *implies* $p_\beta(x) \leq p_\beta(y)$, *there is another lattice seminorm p_α satisfying the following condition. For any directed bounded subset D with* $\sup D = s$, *we have*

$$p_\beta(s) = p_\beta\ (\sup D) \leq \sup \{p_\alpha(d) \mid d \in D\}.$$

Our next two theorems use conditions B and C stated in the following definition. These conditions are somewhat analogous and are inspired by the above seminorm inequality.

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