DEGREE OF SYMMETRY OF STIEFEL MANIFOLDS

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1. Introduction. Let M be a smooth compact connected orientable manifold of dimension m. The degree of symmetry of M as defined by W. Y. Hsiang [2] is the dimension of the largest Lie group which acts differentiably and effectively on M; it is denoted by $N(M)$. It is well known that $N(M) \leq m(m+1)/2$ and equality holds if and only if M is either an m-sphere or a real projective m-space. Manifolds with vanishing first rational Pontrjagin class, $P_1(M; Q)$, have been studied by W. Y. Hsiang. In [3] he proved the following theorem.

THEOREM (Hsiang). *Let M be an orientable manifold of dimension m with $P_1(M; Q) = 0$. If there are generators $v_1, \cdots, v_k$ of $H^*(M; Q)$ such that $v_i \in H^{m_i}(M; Q)$ for $m_i \geq 5$ and $0 \neq v_1 \cup \cdots \cup v_k \in H^m(M; Q)$, then $N(M) \leq \sum_1^k m_i(m_i+1)/2$.*

This theorem is used in [3] to estimate the degree of symmetry of product manifolds. Because the cohomology ring structure of the Stiefel manifolds is known to satisfy the conditions of this theorem, it also gives an estimate of the degree of symmetry of these manifolds which, though not the best to be hoped for, is the best to date. In this paper a much sharper estimate of the degree of symmetry of the Stiefel manifolds is given. The method of calculation is much like that of Hsiang's. The only difference is that Z_2 coefficients and the Steenrod squaring operations are used. This additional structure allows the following generalization of Hsiang's theorem.

THEOREM 1. *Let M be a smooth compact connected orientable manifold of dimension m with $P_1(M; Q) = 0$. Suppose there exist generators $v_j \in H^*(M; Z_2)$, $j = 1, 2, \cdots, l$, of dimensions ≥ 5 and Steenrod operations a_{ij}, $i \in I \subset \{1, 2, \cdots, k_j\}$, with $a_{1j} = S_q^0$ and $a_{ij}v_j \neq 0$ such that*

$$0 \neq \prod_{i,j} a_{ij}v_j \in H^m(M; Z_2).$$

Then

$$N(M) \leq \sum_{i,j} \frac{\dim v_j + 1}{2} (\dim v_j + \dim a_{ij}).$$

Let $O_{n,k}$ denote either the real, complex or quaternionic Stiefel manifold of $(n-k)$-frames in real, complex or quaternionic n-space respectively. If $2k \geq n$, then the Z_2-cohomology and Steenrod operations are known and the structure satisfies the conditions of Theorem 1. Hence a new estimate of

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