SPACES DEFINED BY SEQUENCES OF OPEN COVERS WHICH GUARANTEE THAT CERTAIN SEQUENCES HAVE CLUSTER POINTS

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1. Introduction. In recent years much of the research in point set topology has been devoted to a study of certain generalizations of metrizable spaces. In particular I have in mind developable spaces, wΔ-spaces, semi-stratifiable spaces, first countable and q-spaces, Nagata spaces, σ and Σ-spaces, wM-spaces and others. Each of these classes of spaces can be characterized (or is actually defined) in terms of a sequence of covers which guarantee that certain sequences have cluster points. Such characterizations give a unified approach to these generalized metrizable spaces and suggest problems and new classes of spaces. (See [15] and [14; §5].)

In this paper we study some new classes of generalized metrizable spaces which are defined by a sequence of *open* covers which guarantee that certain sequences have cluster points. In §3 we introduce the class of wN-spaces as a generalization of Nagata spaces and prove that every Hausdorff developable wN-space is metrizable. In §4 we introduce some new classes of first countable and q-spaces and give conditions under which these spaces are metrizable. In §5 we study the relationship between the wM-spaces of Ishii and the spaces introduced in §§3 and 4. Finally in §6 we give some applications of our results to the problem of metrizability.

2. Preliminaries. We begin with some definitions and known results which will be used throughout this paper. Unless otherwise stated no separation axioms are assumed; however regular spaces are always T_1. The set of natural numbers will be denoted by $\mathbf{N}$ and i, j, k and n will denote elements of $\mathbf{N}$.

A space X is *developable* if there is a sequence $\mathcal{G}_1, \mathcal{G}_2, \cdots$ of open covers of X such that for each x in X $\{\text{st}(x, \mathcal{G}_n) : n \text{ in } \mathbf{N}\}$ is a fundamental system of neighborhoods of x. A regular developable space is called a *Moore* space. Bing [4] proved that every collectionwise normal Moore space is metrizable.

Let X be a space, let $\mathcal{G}_1, \mathcal{G}_2, \cdots$ be a sequence of covers of X, and consider the following conditions on $\mathcal{G}_1, \mathcal{G}_2, \cdots$.

(1) If $x_n \in \text{st}^2(p, \mathcal{G}_n)$ for $n = 1, 2, \cdots$, then $\langle x_n \rangle$ has a cluster point.

(2) If $x_n \in \text{st}(p, \mathcal{G}_n)$ for $n = 1, 2, \cdots$, then $\langle x_n \rangle$ has a cluster point.

(3) If x and y are distinct points of X, then there exists n in $\mathbf{N}$ such that $y \notin \text{st}^k(x, \mathcal{G}_n)$.

(4) If x and y are distinct points of X, then there exists n in $\mathbf{N}$ such that $y \notin \text{st}^k(x, \mathcal{G}_n)^-$.

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